Agent-Based Model in Directional-Change Intrinsic Time

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Abstract

We describe an agent-based model where trades happen in event-based time called directional-change intrinsic time. Events defined as the reversal price move of a certain threshold from the local extreme. The price impact of traded volumes is modeled according to the empirically observed squared root impact function typical for the Forex market. The generated time series reproduces statistical properties of foreign exchange rates which are four traditional stylized facts: low auto-correlation of returns, fat-tailed distribution of returns, aggregational normality, price jumps scaling law; and a new method: overshoot scaling law which is omnipresent feature of all liquid markets and states that the expected length of overshoots is equal to the length of the corresponding directional-change threshold.

1   Introduction

Due of the large trading volumes and enumerable list of participants, the Foreign Exchange (FX) market is one of the biggest financial systems where agent-based models were extensively applied for its analysis. According to the Bank for International Settlements, daily trading volume in the FX market increased to an average of \$5.3 trillion in 2013[1]. This volume is generated by the enormous number of transactions made by individual and institutional traders. A proper understanding of mechanisms of such tremendous financial system is crucial for designing risk management tools and to be able to foresee impacts of any political, environmental or technical changes on the health of the system. Since according to the efficient-market hypothesis all relevant to the financial world information is reflected in the prices of various assets the financial time series are mostly used as the main object of a study. Groups of works have been done on the search for fundamental properties of various financial markets embedded in the large amount of data available for researchers today. Thus, Bollerslev and Melvin (1994) used more than 300,000 quotes in an empirical analysis of the bid-ask spread and how it is related to the exchange rate uncertainty; Danielsson and de Vries (1997) and Dacorogna et al. (2001) used high-frequency data to estimate fat tail of exchange rate returns; Kozhan and Salmon (2012) used dataset of market and limit orders to analyze how the information contained in order books could be exploited in simple trading schemes. In one way or

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[1]www.bis.org/publ/rpfx13fx.pdf

another the mentioned in the given research works data represents aggregated behaviour of all agents involved in the trading in the market. To find the exact relation between the behaviour of agents and observed in reality market phenomenon a wide range of agent-based models was proposed. In general the models try to replicate evolving behavior of real market participants (for example, Ehrentreich (2007) analyze whether artificially intelligent agents would converge to the homogeneous rational expectations equilibrium or not). Most of the models serve as complex systems, populated by a large number of independent and heterogeneous agents competing with themselves (NACIRI and TKIOUAT (2016)) and are mostly designed to reproduce and explain phenomenon of real markets, such as bubbles, crashes and regime switches (Samanidou et al. (2007)). Although all models have some specific features designed to imitate given phenomenon one element is always present in any agent-based model: definition of time. In order to describe the interaction between agents and impact of their trading activity on market prices scientists mostly rely on physical time where hours, days or even seasons are used to measure time and periods between events. However, the real market is a complex system with its own endogenous non-constant time flow which speed is dependent on the inhomogeneous frequency of political, social or environmental activity (Guillaume et al. (1997)). This feature leads to non-constant volatility which makes it impossible to equally consider price samples using different equidistant time intervals and lengths of time periods could significantly affect results of experiments. In order to overcome this limitation and to provide a more robust framework able to deal with the price curve independently on the speed of time flow the concept of directional–change intrinsic time was proposed by Guillaume et al. (1997). In this concept events defined as reversal price moves of a certain threshold from local extremes. Only these events allow system's clock tick. Continuous price moves in the same direction of the latest observed directional-change event are called overshoot sections and represent trend components of the price curve. In our work the agent–based model uses the directional change intrinsic time to dissect a price curve into a collection of directional–change and overshoot sections and to identify moments of potential trades. Agents, operating in the directional–change intrinsic time use prices as the only source of information to make decisions on all their trades.

The best way to check whether an agent–based model indeed conforms to the expectations is to compare parameters of generated time series with ones typical for the real financial markets. In the further sections we show that proposed model creates synthetic time series with statistical properties which coincide with 'stylized facts' of real financial time series captured by empirical analysis (Kaldor (1961), Pagan (1996), Gençay et al. (2001), Chakraborti et al. (2009), Kullmann et al. (1999)). Among them: low autocorrelation of returns, fat–tailed distribution of returns, aggregational normality, the price jumps scaling law[2] and overshoot scaling law. The last one is used in addition to the previous four well-known statistical properties usually adopted as benchmarks for agent-based models. It was recently found in a wide range of real high–frequency time series and even in the arithmetic Brownian motion (Dubrulle (1997), Glattfelder et al. (2011)). The overshoot scaling law establishes a relation between the average length of observed overshoots and the corresponding threshold value. The absolute independence of the intrinsic time on the flow of the physical time makes this scaling law very convenient for testing agent-based models. To the extent of our knowledge, it is the first work where statistical properties of the directional–change intrinsic time are used to evaluate an agent-based model.

The rest of this paper is organized as follows; Section 2 illustrates the intrinsic event framework and provides an example of a real price curve dissected into a set of intrinsic events. In Section 3 we describe two main components of our algorithm: the set of artificial agents and the market

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[2]Mathematical relationship between two variables that holds true over multiple orders of magnitude.

response function. A collection of benchmarks used to validate the quality of the generated time series is discussed in Section 4. All obtained results and statistical properties can be found in Section 5. Additionally, in Appendix A we derive the average length of price overshoots for the case of Brownian motion with a constant trend and in Appendix B we propose a pseudo-code of the directional change intrinsic time approach.

2   Intrinsic Time

The volatility of financial time series changes randomly over time (Blattberg and Gonedes (1974), Christie (1982), Scott (1987)) and the number of transactions during holidays or weekends is much lower than during working days or after some unexpected but significant news. Because of this non-homogeneous nature of markets, one can find different time periods with upward or downward price jumps within seconds or very long standstills when there almost no trades. Nevertheless, despite this well–known fact financial market are usually analyzed using equidistant time steps. Mandelbrot and Taylor (1967) and Clark (1973) were one of the first researchers to use the event–based time for modeling and analyzing financial time series. Later Guillaume et al. (1997) defined so–called directional–change intrinsic time where ticks happen as results of the evolution of rising and falling price moves of a certain size (threshold). The breakthrough of this discovery is in the fact that such type of time measure does not relate to exogenous time evolution and only endogenous price moves define the flow of the intrinsic time. The author also presented a scaling-law discovered with help of the new event–based concept and which establishes relation of the chosen intrinsic time thresholds δ with the number of corresponding trend changes $N(\delta)$:

$$N(\delta) = \left(\frac{\delta}{C}\right)^E \tag{1}$$

This scaling law demonstrates how the directional–change intrinsic time is able to get rid of the scalability issue(what is that?) pronounced in the case of physical time.

The main element of directional–change intrinsic time is the changes of a trend of the given size measured in relative values and the length of price moves observed between two consecutive intrinsic events. The concept operates with two states of the trend: upward and downward. A real example of a price curve dissected with intrinsic time is shown on the Figure 1. If the current direction of the trend is upward, then the next **directional–change intrinsic event** will be registered as soon as there is a price reversal of the size equal or greater than the chosen directional change threshold δ measured from the highest observed price since the last directional–change event. Once a new event is observed, an overshoot $\omega(\delta)$ begins and it continues until the next directional–change event. The length of an overshoot defined ex-ante as the distance between the price level of a directional change and the extreme point prior to the next directional–change event.

We initialize the dissection procedure by choosing a starting point called *extreme price* S_{ext} (for example, current price from the market S_{tick}), the relative size of the directional–change price move δ and arbitrary direction of the alternating price move (either $mode_{up}$ for upward or $mode_{down}$ for the downward trend). Each new received price should be compared with the latest registered extreme. If the current mode is $mode_{up}$ ($mode_{down}$) and the newest price S_{tick} is higher (lower) than the extreme price S_{ext} then the S_{ext} takes the value of S_{tick}. Alternatively, the distance the distance between the latest price S_{tick} and the current local extreme S_{ext} should be compared to the size of the threshold δ. If the distance is bigger or equal to δ than the current price is a new directional–change point. At this moment one should change *mode* to the opposite one and reset the local extreme ($S_{ext} = S_{tick}$).

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If after this moment the price continues to move in the same direction, **overshoot intrinsic events** will be registered every time when the size of the overshoot is multiple of the threshold size. There is no limits on the number of overshoot intrinsic events between two consequent directional–changes. As the reader can see from the Figure 1, the minimum size of the overshoot is equal to zero and it happens when the price makes a reversal right after a new directional–change point. A pseudo–code of the algorithm is located in the Appendix B.

From the Figure 1 it is easy to notice that due to the non-homogeneous nature of financial time series intrinsic time ticks more often at periods of high activity and ticks less when the market is relatively quiet. Two latest directional change events are registered only after a weekend (right part of the plot where the exchange rate does not move) simply because during the weekend there were no ticks and no big enough price moves to exceed the directional change threshold δ. Thus, the algorithm dissects a price curve into a set of intrinsic events and does not considers ticks between them. Such behaviour decreases the signal to noise ratio by providing information only about extreme points of trends at different resolutions defined by the size of the given threshold.

There are various advantages of this event–based paradigm of the high–frequency data analysis. Several examples could be found in the following research works:

1. The algorithm was used to show that in markets characterized by various volatility and trends a lot of scaling laws perform in surprisingly similar way (Glattfelder et al., 2011);

2. Several directional–change thresholds, implemented at the same time, were used to describe the price evolution and to compute multi–scale liquidity of given market (Golub et al., 2014);

3. At the moments of high volatility the number of directional–change events growth fast. The number keeps small when the volatility is close to zero. Thus, the approach could be used to estimate the volatility of given time series (Petrov et al., paper in progress).

At any moment of time liquid markets could experience various trends at different scales. In case of a stable trend and a constant threshold δ the approach will return more intrinsic events when the *mode* coincides with the trend and less events otherwise. However, it is possible to modify the original algorithm in such a way that the trend will have no impact on the number of intrinsic events upward and downward. Two different thresholds could be used to measure the distance between the local extreme and the current price: δ_{up} to register directional–change events upward and δ_{down} for any events which happen within a trend down. We include some theoretical analysis in the Appendix A where we demonstrate how the trend effects the expected size of overshoot sections and, therefore, the number of intrinsic events per period of time in case of different *mode* regimes. As it can be seen from equations 15 and 16, the expected size of overshoot sections is not constant and depends on the trend of the market as well as on its volatility: for example, if the trend is negative, upward overshoots $\omega(\delta_{up})$ are equal to the overshoots down $\omega(\delta_{down})$ only if the threshold $\delta_{up} < \delta_{down}$. Because trends depend on the chosen scale it is impossible to say which pair of thresholds $(\delta_{up}, \delta_{down})$ would compensate the given trend specific to the current period. As a possible solution Golub et al. (2017) used an abstraction based on the inventory as the proxy of the trend. In our work we are creating a trivial model where the artificial agents do not keep track of their inventory but have a wide variety of chosen thresholds which are active all the time. The whole set of used agents and their parameters will be described in the following section.
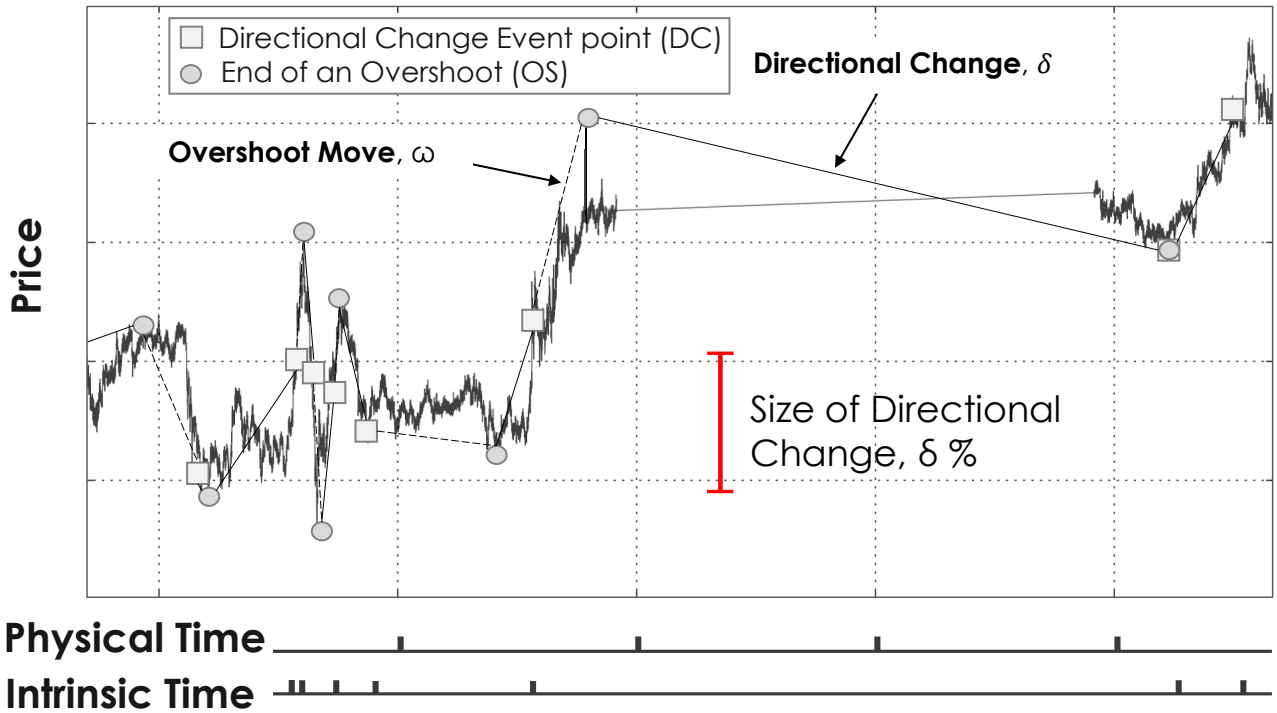
Figure 1: Example of an exchange rate price curve dissected into a set of directional–changes using a symmetric threshold δ. Two types of sections are typical for each consecutive pair of intrinsic events: directional–change (marked by a solid line) and overshot (dashed line). Arbitrary chosen fixed directional–change threshold δ is used for the splitting purposes. The direction of the initial (*mode*) is chosen to be downward. The first (left) grey square marks the first directional-change event which occurs when the price moves downward by δ from the local extreme which for the given mode coincides with the highest observed price (the first grey circle). The mode alternates (upward) as soon as a new directional–change point becomes registered. At this step, the local extreme indicates the smallest observed price since the latest directional –change. The next upward event is registered when a positive return of the size δ happens measured from the local extreme point (minimum price). After this, the *mode* alternates again and the dissection process continues.

3 Structure of the model

Any liquid market could be considered as a combination of only two components: a component representing a group of traders with diverse range of strategies and scales and a component describing impact of their aggregated behavior on the state of the market. In our trivial model we have artificial agents buying and selling fixed volume only at moments of their own intrinsic events and price response function which is a special algorithm generating the next price move (return) as the reaction to the consolidated activity of the agents at the previous price.

The mechanic of the agents is the following: they flip their opened positions with probability P_{flip} at intrinsic events observed in their own intrinsic time determined by the assigned directional change thresholds δ_{up} and δ_{down}. No matter when there are always N_{long} agents who decided to flip their position from sell to buy at the latest observed price and N_{short} agents who decided to be net sell. The trading unit size is fixed and the same for all traders. The difference between the number of long and short agents $\Delta N = N_{long} - N_{short}$ indicates current excess demand or supply and is used to determine the following price change using the volume impact function.

When it is clear that the lack of demand motivates the supply side to reduce the prices and the lack of supply affects the price rise the exact shape of the volume impact function depends on various other factors and is difficult to determine. Several research works have been done on this topic and different models were proposed. A stable and linear impact function was described in the work Kyle (1985) and later Huberman and Stanzl (2004) provided a proof that permanent price impact must indeed be linear while the temporary one can be of a more general form. More sophisticated non-linear price-update function was outlined in several research articles: Hasbrouck (1991), Hausman et al. (1992), Kempf and Korn (1999). For our experiments, we decided to choose the impact function proposed in the relatively recent work Bouchaud (2010). According to Bouchaud, the impact of the trading volume is non-linear and one of the best approximations is the square–root function. Therefore, we endow the market response to the agents' aggregated behaviour with the next net volume impact function:

$$r_n(\Delta N_n) = \lfloor \alpha \cdot \text{sgn}(\Delta N_n) \sqrt{|\Delta N_n|} \rfloor \tag{2}$$

where $r_n(\Delta N_n)$ is one–period return at the step n dependent on the current difference between the total number of buyers and sellers at this step (ΔN_n), α is the parameter which limits the minimum price shift, sgn(.) is the sign function and $\lfloor . \rfloor$ is the floor function. Here we choose the parameter α in such a way that the smallest disbalance between the total number of buyers and sellers will trigger a price return equal to 1: α is equal to $\sqrt{2}/2$.

3.1 Behaviour of the Agents

In real financial markets, market participants have a diverse set of trading strategies: trading in working days or weekends, technical or fundamental analysis, high-frequency trading or holding long-term positions (a survey of US market is provided by Cheung and Chinn (2001)) thus a good agent–based model aiming to mimic the real market should be oriented on the reproducing of similar behavior. Keeping this in mind, we created intrinsic event agents with a unique set of parameters. Since different thresholds lead to the different perception of intrinsic time and various trading behavior our agents have a wide set of unique thresholds which guaranties inhomogeneous patterns of their actions. This was used to resemble various trading activities: with the smallest thresholds, traders register intrinsic events almost at each new price tick (like the real high–frequency traders)

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where with bigger thresholds traders' intrinsic time ticks significantly less often making their behavior similar to the behavior of long–term investors. Each intrinsic event agent makes a decision about his next trading position at moments of intrinsic events location and frequency of which depend on upward and downward directional change thresholds δ_{up} and δ_{down}. In order to diversify the style of their actions, there is no a pair of agents with completely equal parameters. On top of this, the probability P_{flip} determines whether or not an intrinsic agent flips his position at the next intrinsic event which also adds some randomness to the performance of the model. An agent flips his position from long to short by shorting one unit to close his long position and shorting an additional unit to open a short position, in total selling two units. A similar procedure is in place for flipping from a short to a long position, whereas the agent buys two units. An example of intrinsic events registered by a trader with parameters $\delta_{up} = 2, \delta_{down} = 3$ is shown on the Figure 2.
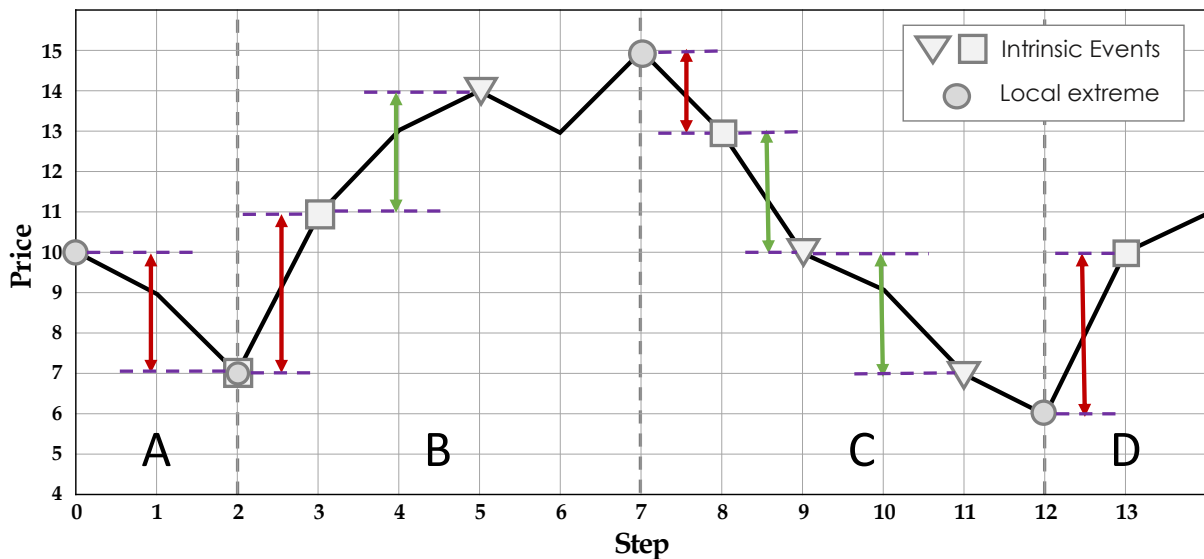
Figure 2: An example of a price curve dissected by a set of points at which an agent with parameters $\delta_{up} = 3$ and $\delta_{down} = 2$ opens a position and makes decisions. The initial price is equal to 10 and the agent's initial *mode* is *down*. Red arrows assign distance measured between local extremes and the following directional change points. Green arrows label price moves which lead to the next overshoot intrinsic event. Letters A, B, C and D are put here to mark four intervals of alternating *mode* states. The agent registers his first directional change intrinsic event as soon as the price goes down from the local extreme (labeled by a circle) by at least 2 points. By definition, the agent opens his first position at this point (step number 2). Then he is waiting for the next event which happens at step 3 after a big price move by four points up from the latest local extreme (coincides with the previous directional change intrinsic event, a grey square). Independently on his previous trading decision, the agent keeps analyzing the price curve. At step 5 the price has made an overshoot move measured from the preceding directional change point and which is equal to δ_{up} (a grey triangle) and means the first overshoot intrinsic event. After this point, the price continues its overshoot move up, but the next event is a directional change at step 8. The next two overshoot intrinsic events happen at steps 9 and 11. The example is concluded by the last directional change intrinsic event point at step 13.

The whole set of used thresholds could be represented by a square grid where on one axis we put values of δ_{up} and on another δ_{down}. On the Figure 3 we demonstrate a part of the grid containing parameters of the intrinsic event agents (traders). Each node there represents a unique trader. The extent to which the agents cover the diversity of various trading patterns is defined by the geometrical size of the grid: L points horizontally and vertically.

The grid can be visually divided into three separate sections. The traders from the region I have upward directional change thresholds larger than the downward one ($\delta_{up} > \delta_{down}$). For these agents,
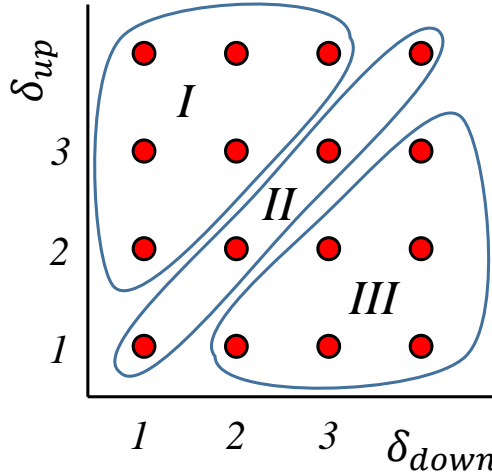
Figure 3: A part of the grid of trading agents. Each point represents an agent defines by a set of unique parameters $\{\delta_{up}, \delta_{down}\}$, where δ_{up} and δ_{down} are the size of upward and downward directional–change thresholds. By numbers I, II and III we mark regions with specific properties: in the region I there are only traders with the upward directional change threshold larger than the downward one ($\delta_{up} > \delta_{down}$), the region II contains "symmetric" agents ($\delta_{up} = \delta_{down}$) and the region III labels all agents with the downward thresholds larger than the upward ones ($\delta_{up} < \delta_{down}$).

two equivalent trends upward and downward are characterized by non–equal number of events. It leads to the fact that in case of zero average trend agents from this region are more eager to buy or sell at moments of downward trends rather than within upward trends (descend supporters). The agents from the region II have equal upward and downward thresholds and are called diagonal agents. The region III marks all agents with upward directional change thresholds smaller than the downward one ($\delta_{up} < \delta_{down}$), so their behavior is the opposite to the traders from the region I (ascent supporters). For each trader from the region I there is a corresponding opposite agent from the region III so the complete set of agents as fully symmetric and thus reflects the balance of different traders in the real financial world. Later in the paper, we will show that the general trend of the generated time series could be directed towards the certain value by activating specific parts of the grid.

The net volume represents the difference between the total supply and demand and is used to compute corresponding price moves aimed to balance the market. In our model, all agents trade with a volume equal to one lot. As a result, the maximum possible price move ΔS_{max} can be observed only when all agents decide to either buy or sell. Thus, the largest price change is determined by the number of agents on the grid and connected to its length L:

$$\Delta S_{max} = \alpha L \sqrt{2} = L \tag{3}$$

In the real world, such big returns are usually interpreted as the market crashes. They do not happen on the daily bases and usually are aftermaths of big numbers of actions accidentally coincided in one critical instant. In our model the probability to observe the maximum price move of size L is minuscule because in order to make it all agents should observe intrinsic events simultaneously, happen to be with the same type of the opened position, and should all decide to flip it. Even just one latest condition has probability P^{L^2} which very rapidly tends to zero with increasing the size of the grid L.

Much more ordinary situation typical for all markets is when general moods of all involved in the trading parties compensate each other thus forming the economic equilibrium. Such states could not last very long and very small market fluctuations enliven further trading activity. Like in the

real world in the proposed agent-based model one can observe zero difference between the number of all buyers and sellers which entails zero net volume and does not cause any price move. Since the agents react only to new price changes we added a trivial random price shift upward or downward with equal probability. The size of such basic price move was chosen to be enough to trigger a new intrinsic event for agents with the smallest thresholds: $\delta = 1$.

In the simulations we use the following parameters: initial price level $S_0 = 0$, the minimum price step $\Delta S_{min} = 1$, $\alpha = \sqrt{2}/2$, the smallest threshold $\delta_{min} = 1$, maximum $\delta_{max} = 50$, step between two consecutive thresholds is 1, total number of trading agents is 2500 and the probability to flip position $P_{flip} = 0.63$. The smallest threshold $\delta = 1$ guaranties that any elementary price move will trigger a new intrinsic event of agents operating in this scale. The size of the probability to flip position coincides with the empirically and theoretically found probability to register a new directional change event prior to an overshoot one. Overall, tick size of generated time series is equal to 1.

The goal of each simulation of the agent-based model is to generate a set of returns which then can be converted to a price curve characterized by the same statistical properties found in the Forex market. In our work, each return generated by the price impact function is defined in logarithmic terms which makes it possible to compare distances between prices in absolute values with the chosen thresholds. In other words, here returns between two given steps m and n ($m > n$) defined as $r(n, m) \approx \log(x_m) - \log(x_n) = S_m - S_n$. Thus, a new intrinsic event happens when the return is bigger or equal than the size of the threshold δ:

$$\delta \geq |r(n, m)(m)| \tag{4}$$

This trivial simplification significantly facilitates all computations in this paper and will be used in the rest of the article.

4 Benchmarks

The main goal of this research work is to check whether an agent-based model build on top of the directional change intrinsic time is capable to generate synthetic time series with statistical properties coherent with ones typical for high-frequency time series from foreign exchange market. Several benchmarks have been chosen to verify the accuracy of the model.

4.1 Traditional methods

One of the well–known evidence about the market microstructures is that price returns at any liquid market do not exhibit significant linear autocorrelation (Arneodo et al., 1996) and in a few minutes, it can be safely assumed to be equal to zero (Cont et al., 1997). This phenomenon is formulated in the "efficient market hypothesis": at such markets prices instantly and fully reflect all available information (Basu (1977)) making it impossible to build a simple trading strategy based on the "statistical arbitrage". Only in very short time intervals when a market is still absorbing a new piece of external information, prices could be characterized by slightly correlated returns. Since this statistical property is one of the most popular stylized facts of all liquid markets we selected it to be one of our benchmarks. Used in the work definition of the autocorrelation function of a time series X with mean μ and variance σ at the given lag τ is

$$R(\tau) = \frac{\mathbb{E}[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2} \tag{5}$$

The second stylized fact used in the analysis is the fat-tailed distribution of returns at a relatively high frequency (more pronounced for intraday values). This fact, also known as excess kurtosis, was pointed out by Benoit Mandelbrot and importance of which was explained in his book Mandelbrot and Hudson (2010) where he points out that despite the wide range of theories build on top of the assumption that the returns could be assumed to be normally distributed the real financial markets have always been much more sophisticated and this discrepancy is a serious flaw of any related financial model. Authors of this work this support his point of view and pay specific attention to this statistical property used as the second benchmark of the agent-based model.

The following simple formula was used to measure the excess kurtosis of the return distribution:

$$k = \frac{\langle (r(t,T) - \langle r(t,T) \rangle)^4 \rangle}{\sigma(T)^4} - 3 \tag{6}$$

where $\sigma(T)^2$ is the variance of the log returns $r(t,T) = x(t+T) - x(t)$. The equation is built in such way that $k = 0$ means an absolutely normal distribution of returns. Brown and Warner (1985) demonstrated that in the stock market kurtosis is usually below 7 and in Cont (2001) it was mentioned that when for SP 500 futures the value is around 16, for Dollar/Swiss Franc futures is approximately 60 and for USD FX rates it is roughly 30 when the time interval is 10 minutes Gençay et al. (2001). It is important to say that though the excess kurtosis is far bigger than zero when time lag is relatively small, it tends to zero as the time lag increases. This fact is usually called the aggregational normality or aggregational Gaussianity and can be accounted for the "mixture of normals" explanation of leptokurtosis (Antypas et al. (2013)). We test generated by the model time series on several time horizons.

The forth traditionally used benchmark is the scaling law which has been constantly reported in several research works: scale-invariance of the absolute price change (return) to the period of time when it occurs (see, for example, Müller et al. (1990), Mantegna and Stanley (1995), Dacorogna et al. (2001)). Even though there is no agreement on the origin of the scaling law (Bouchaud (2001), Farmer et al. (2004), Joulin et al. (2008)) its omnipresence has incentivised scientists to apply it for real financial problems: risk management and volatility modelling (Ghashghaie et al. (1996), Gabaix et al. (2003), Di Matteo (2007)). In our work we check whether the generated by the agent-based model time series can be characterised by this power law and it is our forth benchmark[3].

The same notation proposed in the work Glattfelder et al. (2011) was used to validate this scaling law:

$$y = \left(\frac{x}{C}\right)^E, \tag{7}$$

where $y = \exp Y$, $x = \exp X$, $E = B$ and $C = \exp(-A/B)$ since we assume a linear relationship between the response variable Y (for example, average size of a price return) and the fixed variables

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[3]It is very important to pay attention to the fact that most of scaling laws have been found in financial markets considering not only the inner nature of the price curve expressed in intrinsic events but also using properties of the physical time. It is a very complicated question how one could go back from the intrinsic time used as the main engine of the agent-based model in real seconds, days and years. For example, since volatility in the directional change intrinsic time is just a scaling factor of the frequency of events one cannot directly validate such phenomenon like the volatility clustering of a time series generated by an agent-based model without tiding their behavior to fixed discrete points in the physical time. Nevertheless, the most popular approach used to bridge the gap between physical and intrinsic time in agent-based modeling is the assumption that the agents make decisions at equidistant moments of time, for example, every second. In this case, 20 000 000 steps in the intrinsic time would correspond to 231.5 trading days which is very close to a full trading year (252 days). It is important to remember that this is only an assumption which could be done in this work considering the wide variety of the used thresholds.

X (period of time) in double logarithmic scale, or $Y = A + BX$, where A and B are the unknown parameters to be estimated. Thus B defines the slope of the log-log plot and A is the intersection of the y axis.

4.2   The ultimate benchmark

Despite big popularity of the aforementioned benchmarks they all suffer the same drawback: all of them use physical time as the main indicator of scaling. Artificial agents have no real "feeling" of time and have only "signal-reply" logic. In our work we introduce a new benchmark which can be ultimately used to verify any agent-based model independently on the time measure chosen for the model. In the work of Glattfelder et al. (2011) one of all 12 described scaling laws is totally agnostic to real physical time. The law is called the "overshoot scaling law". It is fully based on the concept of the directional change intrinsic time where only relative price moves dictate sequences of events without taking into account time intervals between them. As it was shown in the work Golub et al. (2014) the probability of overshoot $\omega(\delta; \sigma)$ reaching the length l equals $\exp\left(-\frac{l}{\delta}\right)$, i.e.

$$\mathbb{P}(\omega(\delta; \sigma) \geq l) = \exp\left(-\frac{l}{\delta}\right) \tag{8}$$

which reveals the relation between the length of a directional change threshold δ and the average size of corresponding overshoot $\omega(\delta)$: the average overshoot move $\langle\omega(\delta)\rangle$ is approximately equal to the size of the directional change threshold δ independently on the size of δ:

$$\langle\omega(\delta)\rangle \approx \delta \tag{9}$$

Glattfelder et al. (2011) has shown that for the overshoot scaling law the average coefficients E and C across all exchange rates from the Forex market are $E = 1.04$ and $C = 1.06$. We use these finding as the main benchmark of our model.

As a short remark for this section: scaling laws seem to be kind of ubiquitous properties of our world and present at any domain of natural and social phenomena such like physics, biology and social sciences (Andriani and McKelvey (2007)). Thus it is straightforward to take these omnipresent multi–scale properties into account when validating results of time series generated by artificial sets of interacting agents.

5   Results

In this chapter we highlight the main findings of the research work and demonstrate which components of the agent-based model contributes to the stylized typical for real liquid markets. Two types of experiments have been performed: analysis of time series generated by all agents from the grid and the trend divergence from the zero level in case of asymmetric regions activated for the trading.

5.1   Whole grid

An example of 10 price curves generated by the intrinsic event agents with help of the squared root impact function is presented on the Figure 4. The red line represents the average price based on 1000 independent simulations. As a reader can see this line is perfectly horizontal throughout all steps which means the absence of any trend. At the same time, every chosen price curve does not

demonstrate any prevail direction and consist of various intervals with plateaus and sudden jumps thus mimicking features of real Forex market.
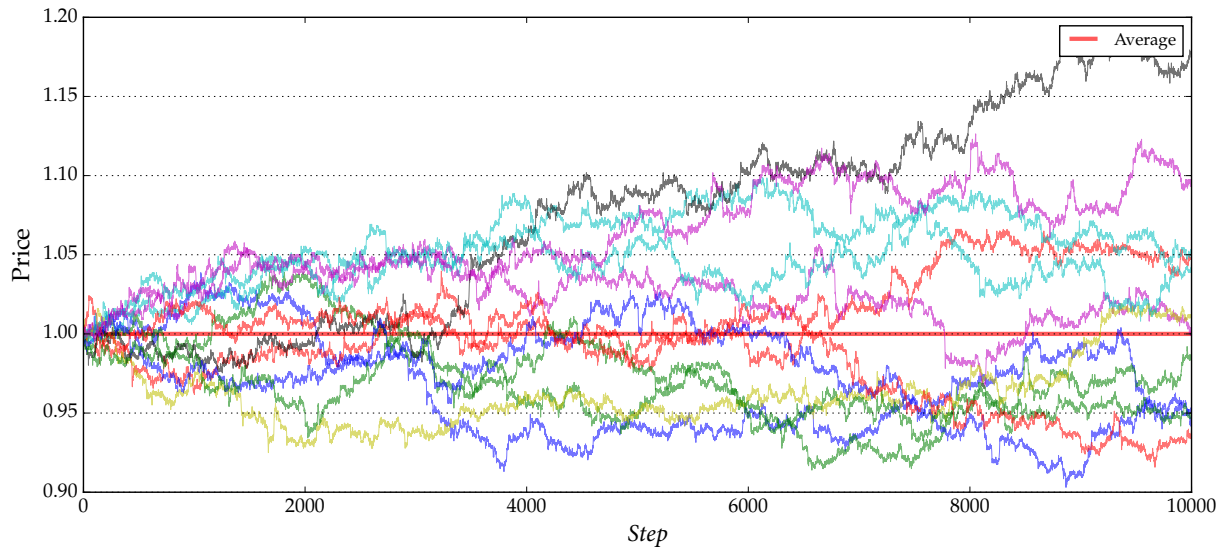
Figure 4: Example of 10 time series generated by the agents from the whole grid 50 by 50 equally distributed points. The presented on this plot price curves were obtained by computing an exponential function of the logarithmic returns generated by the model. For demonstration purposes the generated be the model returns were assumed to be log–returns and the following formula was applied to convert them into the presented curves: $Price = \exp(S_{log}/1000)$

All introduced benchmarks were implemented to validate the artificially generated data sets. Autocorrelation function (ACF) of a long (10 000 000 points) surrogate time series is shown on the Figure 5a. The maximum negative correlation (-0.32) is observed for the lag size equal to 1 step while the rest of the values is significantly less. As expected, the autocorrelation function rapidly decays and becomes indistinguishable from zero already after 10 steps.

The next stylized fact exhibits a long-range slowly decaying autocorrelation function an case of absolute returns. The Figure 5b demonstrates that the bigger lag step, the lower decline of the ACF.

Revealed from a synthetic time series absolute price move scaling law (ΔS as a function of the number of steps) is shown on the Figure 6a. Here C and E are the described in the section 4 characteristics of the scaling law and R is the Pearson product-moment correlation coefficient. The fact that the computed correlation is represented by a straight line on the log-log plot is the most meaningful part of the experiment.

On Figure 7 four plots containing distribution of returns at various lag scales (10, 50, 100 and 1000 steps) are shown. It can be clearly seen that there are persistent fat tails up to lags measured by hundreds of ticks and that they disappear around the level of 1000 steps which is in line with the empirical results (see, for example, Kullmann et al. (1999)). The excess kurtosis value is equal to 2.97 in case of 10 steps lag and just 0.06 when the lag rises to 1000 steps[4]. In addition, on the Figure 8 we present two probability plots which once again confirm the statement.

Finally, we checked whether the overshoot scaling law is also present in the generated time series. The results shown on the Figure 6b are very close to the ones observed at the real foreign exchange market. Moreover, here we also checked two additional versions of the overshoot scaling law: the first

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[4]Even though the excess kurtosis decreases together with the growing lag size in the empirical analysis the value corresponding to small lags is usually much higher (10 and more). However, we found that the size of the grid directly contributes to the excess kurtosis: 50 by 50 points $\longrightarrow k = 2.73$, 100 by 100 points $\longrightarrow k = 3.46$, 200 by 200 points $\longrightarrow k = 6.01$
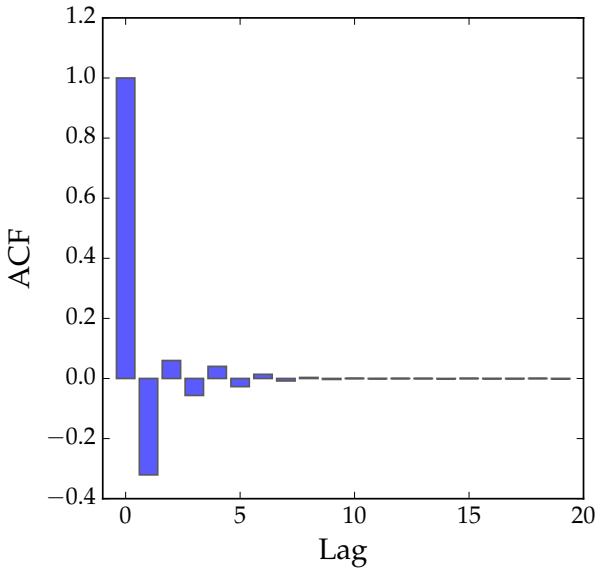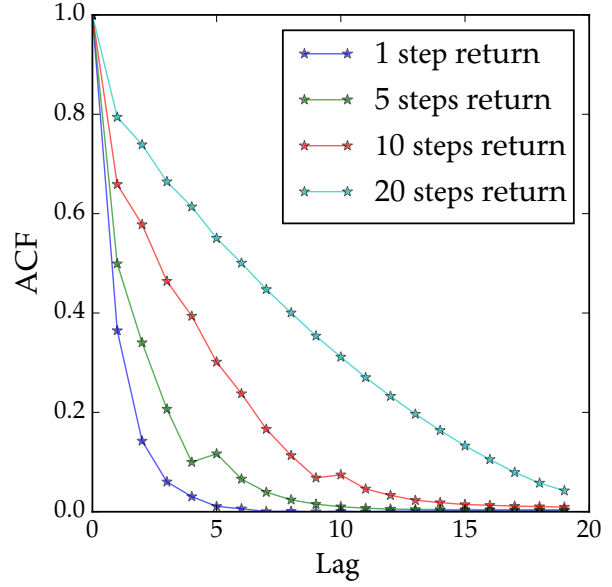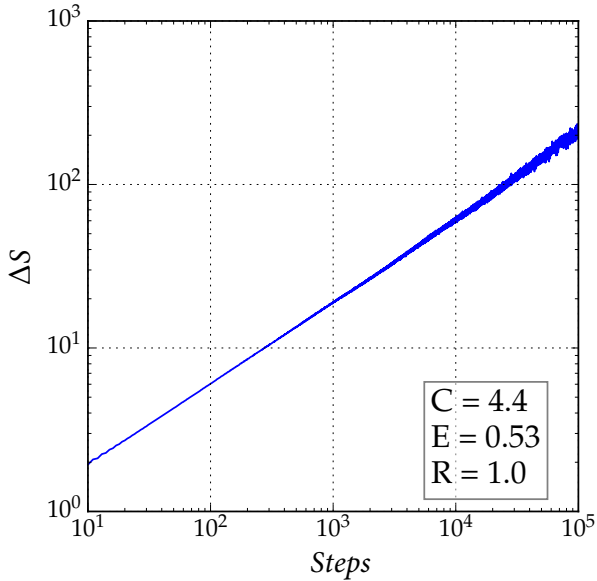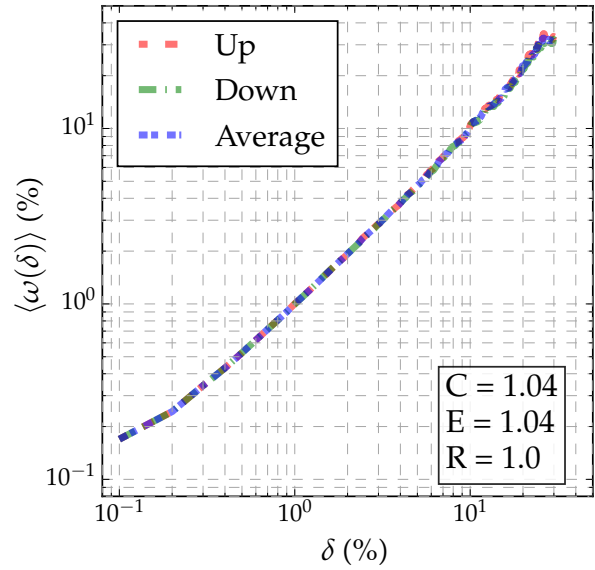
Figure 5: (a) Autocorrelation function of generated time series. Lags are measured in steps. 10 000 000 steps in total. (b) Autocorrelation function of absolute returns.


Figure 6: (a) Average absolute price move as a function of number of steps. (b) Overshoot scaling law for the agents from the entire grid. Parameters on the plot correspond to the average line. Approximation was done for $\delta > 0.3\%$. The same equation presented under the Figure 4 was used to transform thresholds from absolute values to relative ones (percentage). Coefficients of the Up line: $C = 1.04, E = 1.05, R = 1.0$; of the Down: $C = 1.03, E = 1.03, R = 1.0$. The plot is based on 20 000 000 steps.

one is built using only overshoots computed after upward directional changes (red dashed line) and the second is only after the downward ones (green dashed line). As it can be seen on the Figure 6b there is no noticeable difference between all these three lines which once again confirms theoretical

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(a) 10 steps lag.

(b) 50 steps lag.

(c) 100 steps lag.

(d) 1000 steps lag.
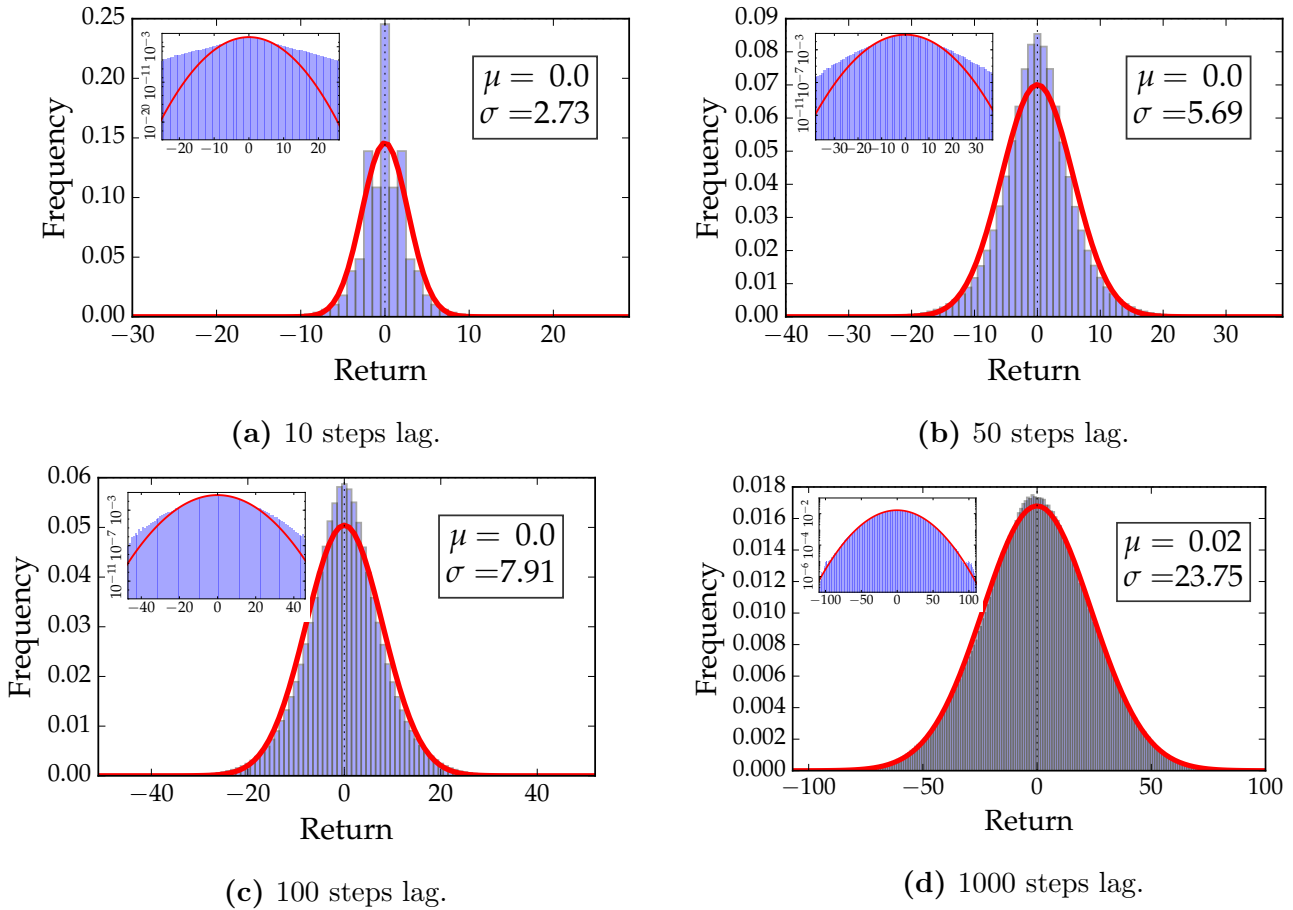
Figure 7: Distribution of returns for generated time series at four different steps lags and their Gaussian approximation (red line). Included sub-plots are the same distributions but in the logarithmic scale.

(a)

(b)
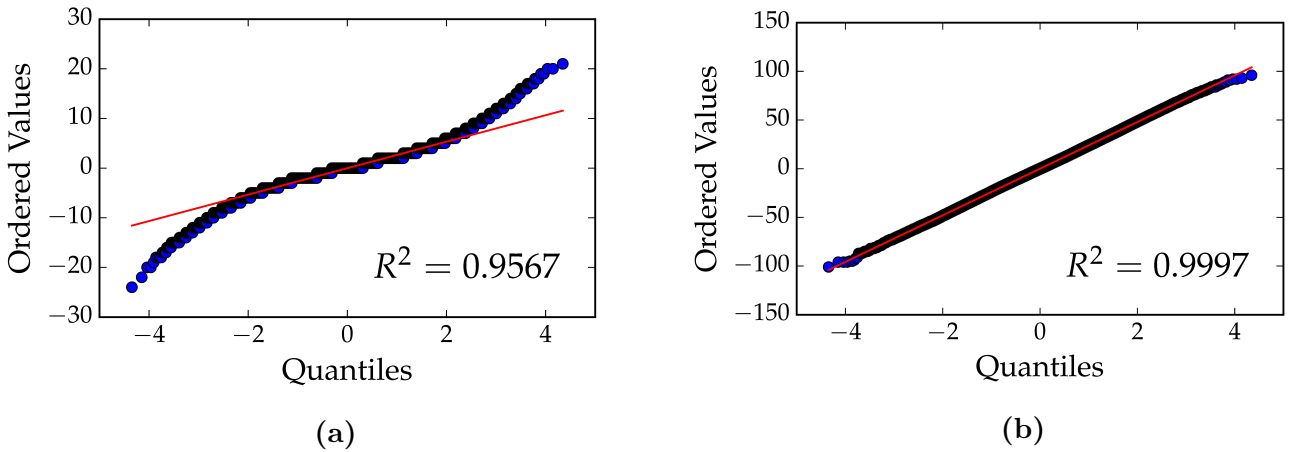
Figure 8: Probability plots created using returns generated by the agent-based model. The total number of steps is 10 000 000. The left-hand side (a) is created for returns computed with the lag equal to 10 steps. The right-hand side (b) contains a probability plot built for returns computed at each 1000 steps.

computations expressed in equations 15 and 16 in the Appendix A.

It is worth mentioning that apart of the square-root volume impact function linear and logarithmic functions have also been tested but they did not manage to replicate the same quality of the stylized

facts. With linear dependence the generated price rapidly fluctuates around the initial level and none of the statistical properties were observed in the data. Statistical properties of time series generated using the logarithmic function reproduces fat-tailed distribution of returns but the overshoot scaling law is still much better reproduced by the originally chosen squared-root function[5].

5.2   Asymmetric regions

A remarkable feature of the presented agent-based model operating in the directional change intrinsic time is that one can easily direct the average price curve upward or downward by tuning set of used intrinsic event agents. Here we put a couple of trivial examples where only agents from the region I or the region III have been selected to trade and as result deviated the average price from the horizontal level. Results of these two experiments are shown on Figure 9.

The permanent trend observed in both experiments could be used to generate realistic time series with predefined characteristics. Several factors affect the slope of the average price curve: total number of agents in the initial grid, fraction of the grid used to generate a time series, selected time interval between two consecutive steps. The precise shape and direction of the trend is a topic for an independent analysis which is out of the scope of this research work.

6   Conclusion

The agent-based model presented in this paper successfully mimics behaviour of real participants of liquid market generating time series with 'stylized facts' observed in the real financial world. The main contribution of the work is in the analysis of a new event–based mechanics underlying the agent–based model: all agents perform in the so–called directional–change intrinsic time where only price moves make time ticks, that is, the intrinsic time is endogenously defined. All five chosen benchmarks have been passed by the agent-based model which let us conclude that real market participants intentionally or unintentionally make trades in a very similar way using their own intrinsic time to reverse their positions. It was also found that non–linear square root impact function empirically observed in real markets is indeed crucial for successful reproducing time series with desired properties.
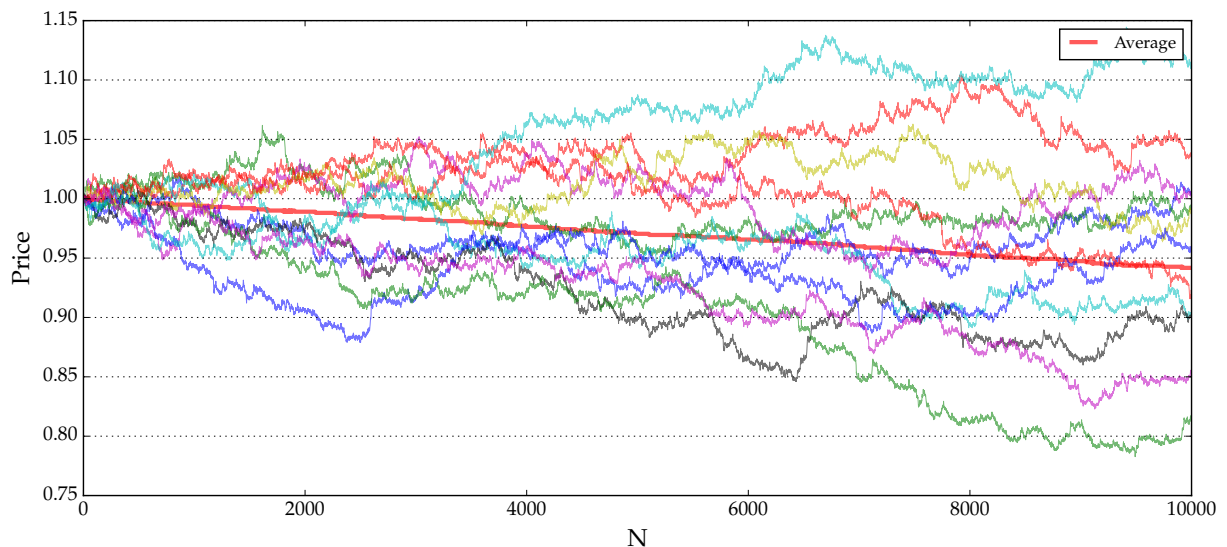
Acknowledgements

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[5]In the case of the linear or logarithmic impact functions the parameters of the overshoot scaling law are noticeably worse than those based on the square root function ($C = 0.86$, $E = 0.95$, $R = 0.16$ and $C = 1.38$, $E = 0.99$, $R = 1.0$ correspondingly versus $C = 1.04$, $E = 1.04$, $R = 1.0$)
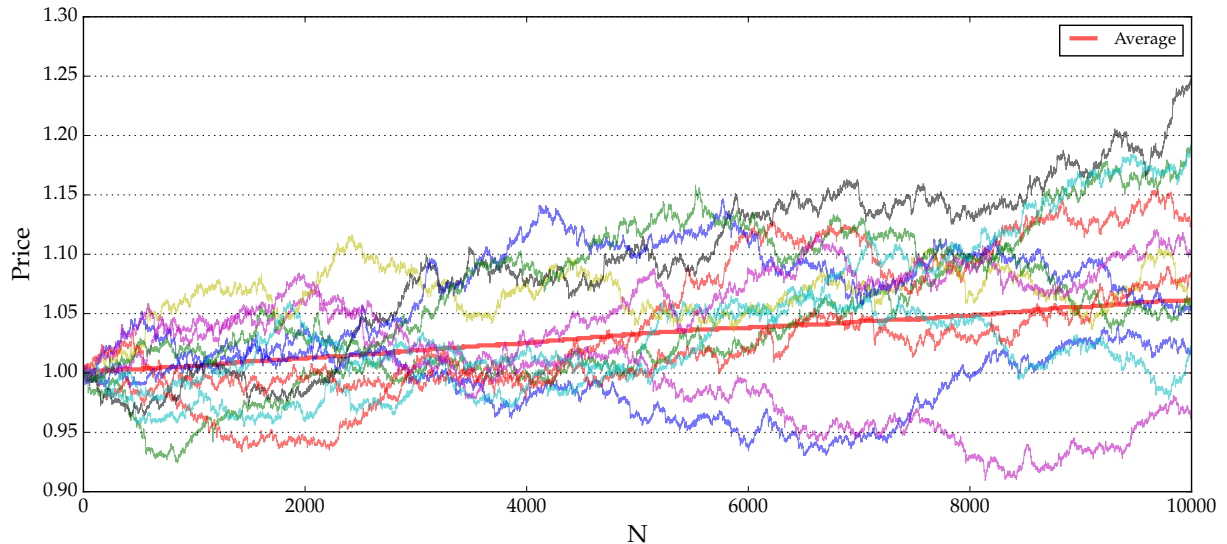
(a)

(b)

Figure 9: (a) Time series generated by the agents from the region I of the initial grid and (b) by the agents from the region III only. Red lines represent average price based on 1000 independent simulations. The initial grid size is 50 by 50 points.

A Overshoot as function of trend

As it was shown in the work of Glattfelder et al. (2011), the average length of an overshoot is approximately equal to the length of the corresponding directional change threshold:

$$\langle \omega(\delta) \rangle \approx \delta \tag{10}$$

This dependence was found not only in the real historical tick data but also for arithmetic Brownian motion without trend. Nevertheless, analysis of Geometrical Brownian Motions with constant trend revealed that the average length of overshoots at not anymore equal to the corresponding size of the threshold and varies together with the size of the trend.

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Simple Brownian Motion with trend μ, volatility σ and a price move dS_t was chosen as the benchmark for the analysis:

$$dS_t = S_t - S_{t-1} = \mu dt + \sigma dW_t \tag{11}$$

Golub et al. (2014) derived the probability of overshoot to reach some fixed value x in case of observing upward overshoot $\omega(\delta_{up})$ and downward overshoot $\omega(\delta_{down})$:

$$\mathbb{P}(\omega(\delta_{up}) \geq x) = \exp\left\{ -\frac{x}{\sigma^2} \cdot \frac{(|\mu| - \mu) + (|\mu| + \mu)\exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}}{1 - \exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}} \right\} \tag{12}$$

$$\mathbb{P}(\omega(\delta_{down}) \geq x) = \exp\left\{ -\frac{x}{\sigma^2} \cdot \frac{(|\mu| + \mu) + (|\mu| - \mu)\exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}}{1 - \exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}} \right\} \tag{13}$$

The expected value of the shown probability equation $\overline{\mathbb{F}}(x) = \mathbb{P}(X \geq x)$ is equal to

$$\mathbb{E}[X] = \int_0^\infty \overline{\mathbb{F}}(x)dx \tag{14}$$

from which the following expected value can be found:

$$\mathbb{E}[\omega(\delta_{up})] = \frac{\sigma^2\left(1 - \exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}\right)}{(|\mu| - \mu) + (|\mu| + \mu)\exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}}, \tag{15}$$

$$\mathbb{E}[\omega(\delta_{down})] = \frac{\sigma^2\left(1 - \exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}\right)}{(|\mu| + \mu) + (|\mu| - \mu)\exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}}. \tag{16}$$

The expected length of the overshoot is just the average of upward and downward expected overshoot, which equals

$$\mathbb{E}[\omega(\delta_{up}, \delta_{down})] = \frac{\sigma^2}{2}\left(\frac{1 - \exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}}{(|\mu| - \mu) + (|\mu| + \mu)\exp\left\{-\frac{2|\mu|\delta_{up}}{\sigma^2}\right\}} + \frac{1 - \exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}}{(|\mu| + \mu) + (|\mu| - \mu)\exp\left\{-\frac{2|\mu|\delta_{down}}{\sigma^2}\right\}} \right) \tag{17}$$

The value depends on four parameters: thresholds δ_{up} and δ_{down}, volatility σ and trend μ. On Figure 10 we demonstrate the dependence of the overshoot length on various trends when volatility is fixed and equal to 1, $\delta = 1$.

It is easy to notice that only in case of zero trend the lengths of upward and downward overshoots coincide with each other, while for any other value of trend when it is not equal to 0 one could observe significant divergence of the curves. The obtained result is quite intuitive: for example, in case of ascending trend price curve more probably will go to the same direction of registered upward directional-change event, therefore the upward overshoot is appreciably longer than the downward one. This observation give the intuition that for each constant trend and volatility at the market there are such thresholds δ_{up} and δ_{down} that the total number directional change events in a given time series will be constant. This property was directly used in our agent-based model when we designed a set of trading agents with different parameters which define their behaviour (Section 3.1).
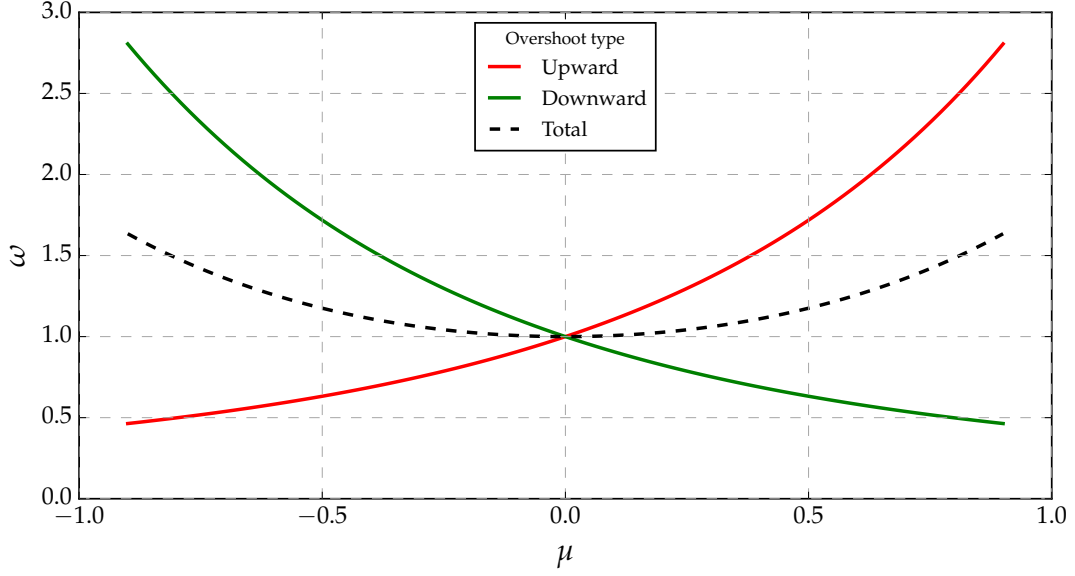
Figure 10: Expected length of overshoots as function of trend computed using equations 15 and 16. The variance σ and threshold values δ are fixed to be equal to 1.

B   Dissection Algorithm

Here by S_{tick} we mark the latest observed price, by S_{ext} the local extreme, *mode* is the current mode of the alternating trend which could be equal either *up* or *down*, δ_{up} and δ_{down} are upward and downward thresholds respectively, S_{IE} is the price at which the latest intrinsic event was observed. The algorithm returns 1 and -1 when the price curve hits the level of an upward and a downward directional change events. 2 and -2 will be returned in case of overshot intrinsic events.

Algorithm 1 Intrinsic Event
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1: **if** *first tick* **then**
2:     $S_{ext} \leftarrow S_{tick}$
3:     $S_{IE} \leftarrow S_{tick}$
4:     **return** 0
5: **else if** *mode* is *up* **then**
6:     **if** $S_{tick} - S_{ext} \geq \delta_{up}$ **then**
7:         $mode \leftarrow down$
8:         $S_{ext} \leftarrow S_{tick}$
9:         $S_{IE} \leftarrow S_{tick}$
10:         **return** 1
11:     **else if** $S_{tick} < S_{ext}$ **then**
12:         $S_{ext} \leftarrow S_{tick}$
13:         **if** $S_{IE} - S_{ext} \geq \delta_{down}$ **then**
14:             $S_{IE} \leftarrow S_{tick}$
15:             **return** -2
16:         **else**
17:             **return** 0
18: **else if** *mode* is *down* **then**
19:     **if** $S_{ext} - S_{tick} \geq \delta_{down}$ **then**
20:         $mode \leftarrow up$
21:         $S_{ext} \leftarrow S_{tick}$
22:         $S_{IE} \leftarrow S_{tick}$
23:         **return** -1
24:     **else if** $S_{tick} > S_{ext}$ **then**
25:         $S_{ext} \leftarrow S_{tick}$
26:         **if** $S_{ext} - S_{IE} \geq \delta_{up}$ **then**
27:             $S_{IE} \leftarrow S_{tick}$
28:             **return** 2
29:         **else**
30:             **return** 0
___

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