

Estimating Nominal Interest Rate Expectations: Overnight Indexed Swaps and the Term Structure*

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Abstract

Financial market participants and policymakers closely monitor the evolution of interest rate expectations. At any given time, the term structure of interest rates contains information regarding these expectations. No-arbitrage dynamic term structure models have regularly been used to estimate interest rate expectations and term premia, but daily frequency estimates of these models fail to accurately capture the evolution of interest rate expectations implied by surveys and financial market instruments. I propose the augmentation of no-arbitrage Gaussian affine dynamic term structure models (GADTSMs) with overnight indexed swap (OIS) rates in order to better estimate the evolution of interest rate expectations and term premia across the whole term structure. I augment the model with 3 to 24-month OIS rates, which provide accurate information about interest rate expectations. The OIS-augmented model that I propose, estimated between January 2002 and December 2016 for the US, generates estimates of the expected path of short-term interest rates, up to the 10-year horizon, that closely correspond to those implied by federal funds futures rates and survey expectations at a range of horizons, and accurately depict their daily frequency evolution. Against these metrics, the interest rate expectation estimates from OIS-augmented models are superior to estimates from existing GADTSMs.

JEL Codes: C32, C58, E43, E47, G12.

Key Words: Term Structure of Interest Rates; Overnight Indexed Swaps; Monetary Policy Expectations; Dynamic Term Structure Model.

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1 Introduction

Financial market participants, researchers and policymakers closely monitor the *daily frequency* evolution of interest rate expectations. To achieve this, they consider a wide range of different financial market instruments and prices. For researchers and policymakers, it is important to attain an accurate real-time measure of the evolution of these expectations in order to form judgements about the appropriateness of policy decisions and to evaluate the effectiveness of existing policies.¹ For investors, understanding future interest rate expectations is important for discounting cash flows, valuing investment opportunities and engaging in profitable trade. At any given time, the term structure of interest rates contains information regarding these expectations. Dynamic term structure models have increasingly been used to estimate and separately identify the dynamic evolution of the expected path of future short-term interest rates and term premia (e.g. [Gagnon, Raskin, Remache, and Sack, 2011](#); [Christensen and Rudebusch, 2012](#); [Lloyd, 2017c](#)),² two components of nominal government bond yields. By imposing no-arbitrage, these models provide estimates of interest rate expectations that are consistent across the term structure, and extend to horizons in excess of what can be accurately imputed from financial market prices directly ([Lloyd, 2017b](#)). However, a popular class of these models — Gaussian affine dynamic term structure models (GADTSMs) — suffers from an identification problem that results in estimates of interest rate expectations that are spuriously stable (e.g. [Bauer, Rudebusch, and Wu, 2012](#); [Kim and Orphanides, 2012](#); [Guimarães, 2014](#)).

Central to the identification problem is an informational insufficiency. Bond yield data is the sole input to an unaugmented GADTSM. These yields provide information of direct relevance to the estimation of the fitted bond yields. Absent additional information, estimates of interest rate expectations are poorly identified as they must also be derived from information contained within the actual bond yields. To do this, maximum likelihood or ordinary least squares estimates of, *inter alia*, the persistence of the (pricing factors derived from the) actual yields must be attained. However, as a symptom of the identification problem, a ‘finite sample’ bias will arise in these persistence parameters when there is insufficient information and a limited number of interest rate cycles in the observed yield data.³ Finite sample bias will result in persistence parameters that are spuriously estimated to be less persistent than they really are and estimates of future short-term interest rates that are spuriously stable.⁴ Because bond yields are highly persistent, the finite sample bias can be severe. Moreover, the severity of the bias is increasing in the persistence of the actual yield data. For daily frequency yields, which

¹See, for example, the literature evaluating the impact of various unconventional monetary policies enacted by central banks since 2007-2008, which uses daily frequency changes in interest rate components to decompose the relative importance of the various transmission channels (for more details, see, [Lloyd, 2017c](#)).

²The term premium represents the compensation investors receive for, *inter alia*, default risk, interest rate risk and illiquidity.

³[Kim and Orphanides \(2012, p. 242\)](#) state that “in a term structure sample spanning 5 to 15 years, one may not observe a sufficient number of ‘mean reversions’.”

⁴This ‘finite sample’ bias is well documented for ordinary least squares estimation of a univariate autoregressive process, where estimates of the autoregressive parameter will be biased downwards, implying less persistence than the true process ([Stock and Watson, 2011](#)). Within GADTSMs, the finite sample bias is a multivariate generalisation of this.

display greater persistence than lower-frequency data, the problem is particularly pertinent.

In this paper, I propose the augmentation of GADTSMs with overnight indexed swap (OIS) rates as an additional estimation input to improve the identification of interest rate expectations and term premia from yields. OIS contracts are over-the-counter traded interest rate derivatives in which two counterparties exchange fixed and floating interest rate payments over its term. A counterparty will enter into an OIS agreement if they expect the payments they swap to exceed those they take on. Thus, OIS rates should reflect the average of investors' expectations of future short-term interest rates. I show that, by providing information for the separate identification of interest rate expectations, OIS-augmentation does tackle the informational insufficiency at the center of the GADTSM identification problem.

Before estimating the OIS-augmented model for the US, I show that OIS rates provide accurate information about investors' expectations of the future short-term interest rate. [Lloyd \(2017b\)](#) verifies that, between January 2002 and December 2016, 1 to 24-month OIS rates accurately reflected expectations of future short-term interest rates in the US, as well as the UK, Eurozone and Japan. Alongside this, I also demonstrate that OIS rates closely align with comparable-horizon survey measures of interest rate expectations.

I then present the OIS-augmented GADTSM, deriving expressions for the OIS pricing factor loadings that explicitly account for the payoff structure in OIS contracts. I estimate the OIS-augmented model using maximum likelihood via the Kalman filter with 3 to 24-month OIS rates and 3-month to 10-year US Treasury yields. The model provides estimates of interest rate expectations and term premia out to a 10-year horizon. To the extent that excess returns on OIS rates can vary on a day-to-day basis, I admit measurement error in the OIS excess returns over time in my OIS-augmented GADTSM. The Kalman filter maximum likelihood setup is well suited to account for this.

This is not the first paper to propose a solution to the GADTSM identification problem. [Kim and Orphanides \(2012\)](#) suggest the augmentation of GADTSMs with survey expectations of future short-term interest rates for the same purpose. They document that, between 1990 and 2003, the survey-augmented model produces sensible estimates of interest rate expectations. [Guimarães \(2014\)](#) shows that, relative to an unaugmented GADTSM, the survey-augmented model provides estimates of interest rate expectations that better correspond with survey expectations of future interest rates and delivers gains in the precision of interest rate expectation estimates. However, I show that estimated interest rate expectations from the OIS-augmented model are superior to the survey-augmented model for the 2002-2016 period.

[Bauer et al. \(2012\)](#) propose an alternative solution, focused on directly resolving the finite sample bias via bias-correction. They document that their bias-corrected estimates of interest rate expectations “are more plausible from a macro-finance perspective” (p. 454) than those from an unaugmented GADTSM. However, as [Wright \(2014\)](#) states, the fact that bias-correction has notable effects on GADTSM-estimated interest rate expectations is merely a *symptom* of the identification problem. Bias-correction does not directly address the identification problem at the heart of GADTSM estimation: the informational insufficiency. Moreover, [Wright \(2014\)](#)

argues that the bias-corrected estimates of future interest rate expectations are “far too volatile” (p. 339). I find that estimated interest rate expectations from the OIS-augmented model are superior to bias-corrected estimates for the 2002-2016 period.

OIS-augmentation is closest in philosophy to survey-augmentation. The GADTSM is augmented with additional information to better identify the evolution of interest rate expectations. However, OIS-augmentation differs in a number of important respects, which help to explain its superior performance *vis-à-vis* survey-augmentation. Primarily, although survey forecasts do help to address the informational insufficiency problem, they are ill-equipped for the estimation of daily frequency expectations. Survey forecasts of future interest rates are only available at a low frequency: quarterly or monthly, at best. Thus, survey forecasts are unlikely to provide sufficient information to accurately identify the daily frequency evolution of interest rate expectations. Moreover, the survey forecasts used by [Kim and Orphanides \(2012\)](#) and [Guimarães \(2014\)](#) correspond to the expectations of professional forecasters and not necessarily those of financial market participants.

OIS rates offer significant advantages over survey expectations for the daily frequency estimation of GADTSMs. Most importantly, OIS rates are available at a daily frequency, so provide information at the same frequency at which interest rate expectations are estimated. Secondly, OIS rates are formed as a result of actions by financial market participants, so can be expected to better reflect their expectations of future short-term interest rates. Third, the information in survey forecasts is limited in comparison to the expectational information contained in OIS rates. Survey forecasts typically provide information about expected future short-term interest rates for a short time period in the future.⁵ In contrast, there exists a term structure of OIS contracts that can be used to infer the evolution of investors’ interest rate expectations from now until a specified future date. The horizon of these OIS contracts corresponds exactly to the horizon of nominal government bonds.

Away from the GADTSM-literature, OIS rates are increasingly being used to infer investors’ expectations of future monetary policy (e.g. [Christensen and Rudebusch, 2012](#); [Woodford, 2012](#); [Bauer and Rudebusch, 2014](#); [Lloyd, 2017c](#)). These authors attribute daily changes in OIS rates to changes in investors’ expectations of future short-term interest rates. [Lloyd \(2017b\)](#) formally studies the empirical performance of OIS rates as financial market-based measures of investors’ interest rate expectations. [Lloyd \(2017b\)](#) first compares the *ex post* excess returns on US OIS contracts with portfolios of federal funds futures contracts spanning the same maturity. Federal funds futures are widely used as market-based measures of monetary policy expectations at near-term horizons, and [Gürkaynak, Sack, and Swanson \(2007b\)](#) document that they dominate a range of other financial market instruments in forecasting the future path of short-term interest rates at horizons out to six months.⁶ For the 2002-2016 period, [Lloyd \(2017b\)](#) finds that 1 to

⁵For example, the Federal Reserve Bank of Philadelphia’s *Survey of Professional Forecasters* provides expectations of the average 3-month T-Bill rate during the current quarter, and the first, second, third and fourth quarters ahead.

⁶[Gürkaynak et al. \(2007b\)](#) compare the predictive power of federal funds futures to term federal funds loans, term eurodollar deposits, eurodollar futures, Treasury bills and commercial paper of comparable maturities. They do not compare federal funds futures rates with OIS rates.

11-month US OIS rates provide measures of monetary policy expectations that are as good as comparable-maturity federal funds futures rates. [Lloyd \(2017b\)](#) also assesses the empirical performance of OIS rates in the US, at longer maturities, and the UK, Japan and the Eurozone. [Lloyd \(2017b\)](#) concludes that UK, Japanese and Eurozone OIS rates provide similarly good measures of interest rate expectations out to the 2-year horizon, implying that the method proposed in this paper is widely applicable in other countries (see [Lloyd, 2017a](#), for global applications of the model).

OIS rates offer a further advantage over federal funds futures as a measure of interest rate expectations in a GADTSM-setting. The horizon of OIS contracts corresponds exactly to the horizon of the bond yield data used in GADTSMs. The horizon of a federal funds futures contract is a single month in the future, beginning on the first and ending on the last day of a specified month. Thus, OIS contracts provide a richer source of information with which to identify expected future short-term interest rates along the term structure.

I document that the OIS-augmented model accurately captures investors' expectations of future short-term interest rates out to the 10-year horizon. The in-sample model estimates of interest rate expectations co-move closely with federal funds futures rates and survey expectations of future short-term interest rates at horizons where such a comparison is possible. In these dimensions, the OIS-augmented model is superior to three other GADTSMs: (i) the unaugmented model, which only uses bond yield data to estimate both actual yields and interest rate expectations; (ii) the bias-corrected model of [Bauer et al. \(2012\)](#); and (iii) the survey-augmented model.⁷ The OIS-augmented model is also best able to capture qualitative daily frequency movements in interest rate expectations implied by financial market instruments. Moreover, unlike the other models, the interest rate expectations implied by the OIS-augmented model obey the zero lower bound for the US, despite the fact that additional restrictions are not imposed to achieve this. This represents an important contribution in the light of recent computationally burdensome proposals for term structure modelling at the zero lower bound (e.g. [Christensen and Rudebusch, 2013a,b](#)).

The remainder of this paper is structured as follows. [Section 2](#) describes OIS contracts and the accuracy of OIS-implied interest rate expectations. [Section 3](#) lays out the unaugmented arbitrage-free GADTSM, before describing the identification problem and 'finite sample' bias with direct reference to the model parameters. [Section 4](#) presents the OIS-augmented model. [Section 5](#) documents the data and estimation methodology. [Section 6](#) presents the results, documenting the superiority of the OIS-augmented model as a measure of interest rate expectations. [Section 7](#) concludes.

⁷For the most direct comparison to the OIS-augmented model, I estimate the survey-augmented model using the algorithm of [Guimarães \(2014\)](#) which uses the same [Joslin et al. \(2011\)](#) identification restrictions as the OIS-augmented model, as opposed to the [Kim and Wright \(2005\)](#) survey-augmented model that applies the [Kim and Orphanides \(2012\)](#) identification algorithm, first proposed in [Kim and Orphanides \(2005\)](#). [Lloyd \(2017c\)](#) shows that the [Kim and Wright \(2005\)](#) model performs worse than the OIS-augmented decomposition.

2 Overnight Indexed Swaps

An overnight index swap (OIS) is an over-the-counter traded interest rate derivative with two participating agents who agree to exchange fixed and floating interest rate payments over a *notional* principal for the life of the contract. The floating leg of the contract is constructed by calculating the accrued interest payments from a strategy of investing the notional principal in the overnight reference rate and repeating this on an overnight basis, investing principal plus interest each time. The reference rate for US OIS contracts is the effective federal funds rate. The ‘OIS rate’ represents the fixed leg of the contract. For vanilla US OIS contracts with a maturity of one year or less, money is only exchanged at the conclusion of the OIS contract. Upon settlement, only the net cash flow is exchanged between the parties.⁸ That is, if the accrued fixed interest rate payment exceeds the floating interest payment, the agent who took on the former payments must pay the other at settlement. Importantly, there is no exchange of principal at any time for OIS contracts of all maturities.

Given its features, changes in OIS rates can reasonably be associated with changes in investors’ expectations of future overnight interest rates over the horizon of the contract (Michaud and Upper, 2008). OIS contracts should contain only very small excess returns. Notably, because OIS contracts do not involve any initial cash flow, their liquidity premia will be small. Additionally, because OIS contracts do not involve an exchange of principal, their associated counterparty risk is small. Because many OIS trades are collateralised, credit risk is also minimised (Tabb and Grundfest, 2013, pp. 244-245). Unlike many LIBOR-based instruments, OIS contracts have increased in popularity amongst investors following the 2007-2008 financial crisis (Cheng, Dorji, and Lantz, 2010).

2.1 Excess Returns on Overnight Indexed Swaps

To assess the magnitude of the excess returns within OIS rates, I present a mathematical expression for this quantity. Let $i_{t,t+n}^{OIS}$ denote the annualised n -month OIS rate, the fixed interest rate in the swap, quoted in month t . Let $i_{t,t+n}^{FLT}$ denote the annualised *ex post* realised value of the floating leg of the same swap contract. From the perspective of an agent who swaps fixed interest rate payments for the floating rate over the notional principal x , the net cash flow received is $(i_{t,t+n}^{OIS} - i_{t,t+n}^{FLT}) \times x$.

The floating leg of the contract $i_{t,t+n}^{FLT}$ is calculated by considering a strategy in which an investor borrows the swap’s notional principal x , invests in the overnight reference rate and repeats the transaction on an overnight basis, investing principal plus interest each time. Let the contract trade day be denoted t_{1-s} , where s denotes the ‘spot lag’ of the contract in days. US OIS contracts have a two-day spot lag $s = 2$, so the trade date is denoted t_{-1} .⁹ Suppose that the n -month (N -day) contract matures on the day t_N in the calendar month $t + n$. Then, the floating leg is calculated based on the realised effective federal funds rate — the floating

⁸For OIS contracts with maturity in excess of one year, net cash flows are exchanged at the end of every year.

⁹That is, calculation of the payments to be made under the floating and fixed legs of the swap does not commence until two days after the contract was agreed.

overnight reference rate for US OIS contracts — on days t_1, t_2, \dots, t_N , where the effective federal funds rate on the day t_j is ffr_j . Following market convention, the mathematical expression for the floating leg of an n -month OIS contract, purchased on day t_{-1} in month t is:¹⁰

$$i_{t,t+n}^{FLT} = \left(\left[\prod_{j=1}^N (1 + \gamma_j ffr_j) \right] - 1 \right) \times \frac{360}{N} \quad (1)$$

where γ_j is the accrual factor of the form $\gamma_j = D_j/360$, where D_j is the day count between the business days t_j and t_{j+1} .¹¹ To compare this floating leg to the fixed leg $i_{t,t+n}^{OIS}$, which is reported on an annualised basis, $i_{t,t+n}^{FLT}$ is a multiple of $360/N$ in (1).¹²

From the perspective of the agent who swaps fixed for floating interest rate payments, $(i_{t,t+n}^{OIS} - i_{t,t+n}^{FLT}) \times x$ represents the payoff of a zero-cost portfolio.¹³ Thus, in accordance with the terminology of [Piazzesi and Swanson \(2008\)](#), the ‘*ex post* realised excess return’ on the n -month OIS contract purchased in month t is:

$$rx_{t,t+n}^{ois} = i_{t,t+n}^{OIS} - i_{t,t+n}^{FLT} \quad (2)$$

Under the expectations hypothesis, the fixed leg of the OIS contract must equal the *ex ante* expectation of the floating leg:

$$i_{t,t+n}^{OIS} = \mathbb{E}_t [i_{t,t+n}^{FLT}] \quad (3)$$

If the *ex post* realised excess return in (2) has zero mean, the *ex ante* forecasting error under the expectations hypothesis also has zero mean, supporting the proposition that the n -month OIS rate provides an accurate measure of investors’ expectations of future short-term interest rates. In constructing the OIS-augmented GADTSM, I assume that the included OIS tenors satisfy (3), motivating the estimation of *ex post* realised excess returns on OIS contracts of various maturities to test this assumption.

[Lloyd \(2017b\)](#) estimates average *ex post* realised excess returns on US OIS contracts using regressions of the following form:

$$rx_{t,t+n}^{ois} = \alpha^{(n)} + \varepsilon_{t,t+n} \quad (4)$$

for the following maturities: 1, 2, ..., 11 months; 1 year; 15, 18, 21 months; 2 and 3 years. [Lloyd \(2017b\)](#) demonstrates that, for the 2002-2016 sample as a whole, 6 to 21-month US OIS contracts have statistically insignificant average *ex post* excess returns. The average *ex post* excess returns on 1 to 5-month US OIS contracts are significant at the 10% level, but are small

¹⁰See both [Cheng et al. \(2010\)](#) and [OpenGamma \(2013\)](#).

¹¹For example, on a week with no public holidays, the day count D_j will be set to 1 on Monday to Thursday, 3 on Friday, and 0 on Saturday and Sunday. The day count is divided by 360, and not 365, in accordance with the quoting convention of the US market ([OpenGamma, 2013](#)).

¹²This, again, is in accordance with the US market quoting conventions. The fixed and floating legs of US OIS contracts are quoted according to the *Actual 360* market convention ([OpenGamma, 2013](#), p. 6).

¹³Formally, this portfolio involves borrowing x at the floating overnight index rate at day t_1 and rolling over the borrowing to day t_N (resulting in the total floating rate payment $i_{t,t+n}^{FLT}$), while investing the x borrowed on day t_1 in the fixed interest rate $i_{t,t+n}^{OIS}$ for N days.

— less than 7 basis points. Notwithstanding this, [Lloyd \(2017b\)](#) shows that, when accounting for 2007-2008 money market turmoil and the US monetary policy loosening of 2008 that was unexpected *ex ante*, the average *ex post* excess returns on 1 to 24-month US OIS contracts are statistically insignificant. That is, 1 to 24-month OIS rates provide accurate measures of investors’ interest rate expectations, conforming to the expectations hypothesis, as stated in (3), and verifying an important identifying assumption of the OIS-augmented GADTSM.

2.2 OIS Rates and Survey Expectations

To further illustrate that OIS rates provide accurate information about expectations of the future short-term interest rates, I compare OIS rates with survey expectations.

Figure 1 plots daily 3, 6 and 12-month OIS rates between January 2002 and December 2016 against both the daily frequency *ex post* realised floating leg of the swap and the quarterly frequency survey expectations of the future short-term nominal interest rate over the corresponding-horizon. I construct approximations of survey forecasts for the average 3-month US T-Bill rate for each of the horizons using data from the *Survey of Professional Forecasters* (SPF) at the Federal Reserve Bank of Philadelphia.¹⁴ The survey is published every quarter and reports the median forecasters’ expectations of the average 3-month T-Bill rate over specified time periods: the current quarter $\bar{i}_{t|t}^{3m,sur}$; and the first $\bar{i}_{t+1|t}^{3m,sur}$, second $\bar{i}_{t+2|t}^{3m,sur}$, third $\bar{i}_{t+3|t}^{3m,sur}$ and fourth $\bar{i}_{t+4|t}^{3m,sur}$ quarters subsequent to the current one, where t denotes the current quarter. To construct the survey forecast approximations in figure 1, I first calculate the implied expectations of the average 3-month T-Bill rate over the remainder of the current quarter using the realised 3-month T-Bill rate over the current quarter up to the survey submission deadline date and the median survey expectation for the average 3-month T-Bill rate for the current quarter $\bar{i}_{t|t}^{3m,sur}$, exploiting the fact that the survey deadline dates lie approximately halfway through the ‘current’ quarter.¹⁵ Using this and the longer-horizon survey expectations, I then calculate geometric weighted averages of survey forecasts from the SPF (see appendix B). I use a geometric weighting scheme to allow comparison with the geometric payoff structure of OIS contracts.¹⁶

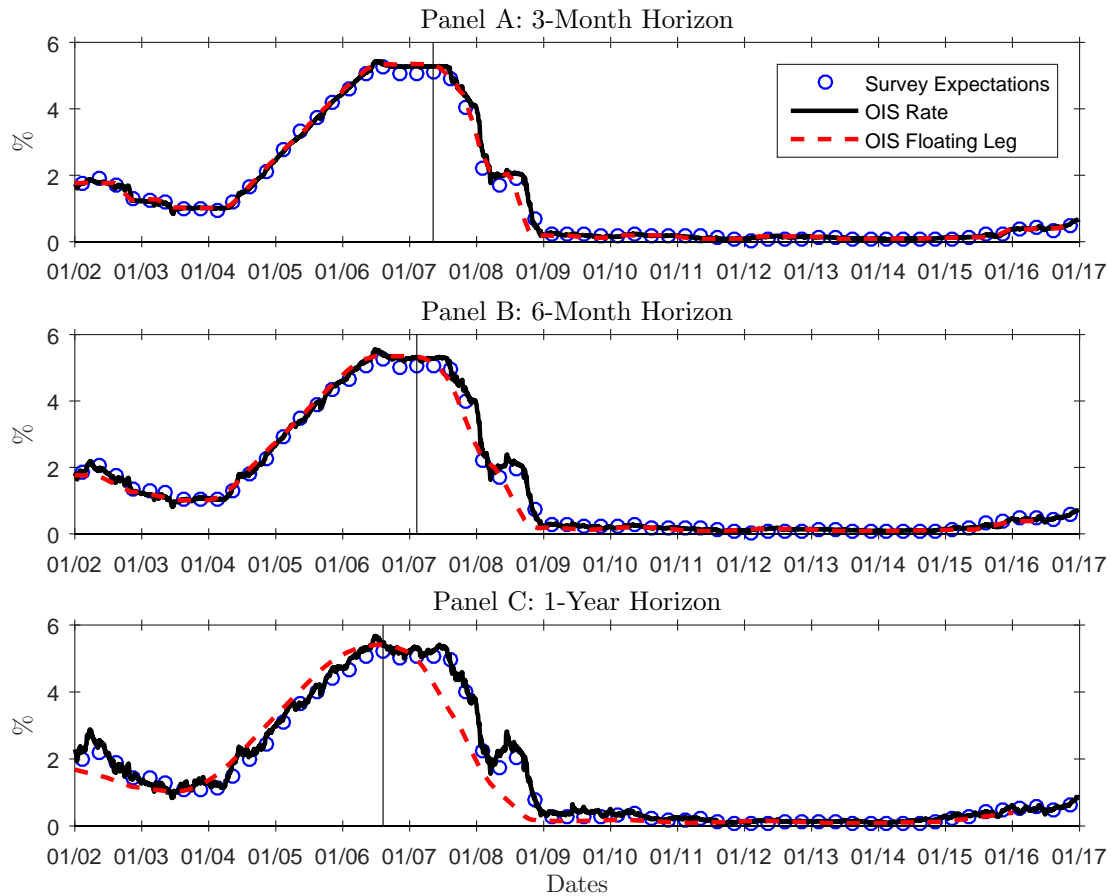
Figure 1 plots survey expectations on submission dates, and demonstrates that survey and OIS-implied interest rate expectations co-moved closely between 2002 and 2016. The difference between the OIS rate i_t^{OIS} (solid black line) and the *ex post* realised floating leg i_t^{FLT} (dashed red line) graphically depicts the excess return defined in (2). Visual inspection of figure 1 confirms the formal results from [Lloyd \(2017b\)](#): the OIS rate closely co-moves with the *ex post* realised path of the floating leg of the contract. The most notable deviation of the two quantities

¹⁴See Appendix A for a detailed specification of data sources. [Guimarães \(2014\)](#) uses survey forecasts from the *Survey of Professional Forecasters* in his estimation of the US term structure of interest rates.

¹⁵For example, the deadline date for the 2013 Q1 survey was February 11th 2013.

¹⁶There are two caveats to this comparison which help to explain small differences between survey expectations and OIS rates. First, the expectational horizons of OIS rates and the T-Bill expectations do not exactly correspond, because the latter also reflect 3-month T-Bill rate expectations 1.5 months beyond the horizon, which reflect expected developments up to 4.5 months beyond the horizon. Second, 3-month T-Bill rates are on a discount basis, whereas OIS rates include expectations of interest rates on a yield basis.

Figure 1: US OIS Rates and Corresponding *Ex Post* Realised Floating Rates, and Survey Expectations



Note: Daily OIS rates from *Bloomberg*. Daily *ex post* realised floating legs of the swaps calculated using equation (1). Survey expectations are from the *Survey of Professional Forecasters*. January 2002 to December 2016. The survey forecast, at each horizon, is attained by constructing the geometric weighted average of the median response of forecasters relating to their expectation of the average 3-month T-Bill rate over the relevant periods (see appendix B). Survey expectations are plotted on the forecast submission deadline date for each quarter. See appendix A for detailed data source information. Vertical lines in each panel are plotted 3, 6 and 12 months prior to August 9, 2007 respectively, the date BNP Paribas froze funds citing US sub-prime mortgage sector problems.

occurs in 2007-2008, coinciding with the financial turmoil that erupted in this period.¹⁷ As the 2008 Federal Reserve policy easing was *ex ante* unanticipated, there is no reason to expect it to be reflected in *ex ante* expectations of future interest rates, explaining the difference in the quantities at this time. Similarly, there is a small difference between the 1-year OIS rate and the realised floating rate during 2002, an artefact of unexpected US monetary policy loosening in response to the 2001 recession.

¹⁷The vertical lines in figure 1 denote the time period 3, 6 and 12 months prior to August 9, 2007 respectively, the date BNP Paribas froze funds citing US sub-prime mortgage sector problems.

3 Term Structure Model

This section presents the discrete-time GADTSM that is commonplace in the literature (e.g. [Ang and Piazzesi, 2003](#); [Kim and Wright, 2005](#)) and describes the identification problem, which arises from the estimation of unaugmented GADTSMs, with direct reference to the model's parameters. Since the focus of this paper is on the identification of interest rate expectations and term premia at a daily frequency, hereafter t is a *daily* time index.¹⁸

3.1 Unaugmented Model Specification

The discrete-time GADTSM builds on three key foundations. First, there are K pricing factors \mathbf{x}_t (a $K \times 1$ vector), which follow a first-order vector autoregressive process under the actual probability measure \mathbb{P} :

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1} \quad (5)$$

where $\boldsymbol{\varepsilon}_{t+1}$ is a stochastic disturbance with the conditional distribution $\boldsymbol{\varepsilon}_{t+1}|\mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$; $\mathbf{0}_K$ is a $K \times 1$ vector of zeros; and \mathbf{I}_K is a $K \times K$ identity matrix. $\boldsymbol{\mu}$ is a $K \times 1$ vector and $\boldsymbol{\Phi}$ is a $K \times K$ matrix of parameters. $\boldsymbol{\Sigma}$ is a $K \times K$ lower triangular matrix, which is invariant to the probability measure.

Second, the one-period short-term nominal interest rate i_t is assumed to be an affine function of the pricing factors:

$$i_t = \delta_0 + \boldsymbol{\delta}'_1 \mathbf{x}_t \quad (6)$$

where δ_0 is a scalar and $\boldsymbol{\delta}_1$ is a $K \times 1$ vector of parameters.

Third, no-arbitrage is imposed. The pricing kernel M_{t+1} that prices all assets when there is no-arbitrage is of the following form:

$$M_{t+1} = \exp\left(-i_t - \frac{1}{2}\boldsymbol{\lambda}'_t \boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t \boldsymbol{\varepsilon}_{t+1}\right) \quad (7)$$

where $\boldsymbol{\lambda}_t$ represents a $K \times 1$ vector of time-varying market prices of risk, which are affine in the pricing factors, following [Duffee \(2002\)](#):

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \quad (8)$$

where $\boldsymbol{\lambda}_0$ is a $K \times 1$ vector and $\boldsymbol{\Lambda}_1$ is a $K \times K$ matrix of parameters.

The assumption of no-arbitrage guarantees the existence of a risk-adjusted probability measure \mathbb{Q} , under which the bonds are priced ([Harrison and Kreps, 1979](#)).¹⁹ Given the form of the market prices of risk in (8), the pricing factors \mathbf{x}_t also follow a first-order vector autoregressive

¹⁸The model can be estimated at lower frequencies, with the label for t changing correspondingly. Appendix [F.3](#) presents a comparison of models estimated at a monthly frequency. The results from monthly frequency estimation are similar to those from daily frequency estimation.

¹⁹The risk-adjusted probability measure \mathbb{Q} is defined such that the price V_t of any asset that does not pay any dividends at time $t+1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)V_{t+1}]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the risk-adjusted probability measure \mathbb{Q} .

process under the risk-adjusted probability measure \mathbb{Q} :

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} \quad (9)$$

where:²⁰

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_0, \quad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma} \boldsymbol{\Lambda}_1.$$

and $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}$ is a stochastic disturbance with the conditional distribution $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} | \mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$.

Bond Pricing Since M_{t+1} is the nominal pricing kernel that prices all nominal assets in the economy, the gross one-period return R_{t+1} on any nominal asset must satisfy:

$$\mathbb{E}_t [M_{t+1} R_{t+1}] = 1 \quad (10)$$

Let $P_{t,n}$ denote the price of an n -day zero-coupon bond at time t . Then, using $R_{t+1} = P_{t+1,n-1}/P_{t,n}$, (10) implies that the bond price is recursively defined:

$$P_{t,n} = \mathbb{E}_t [M_{t+1} P_{t+1,n-1}] \quad (11)$$

Alternatively, with no-arbitrage, the price of an n -period zero-coupon bond must also satisfy the following relation under the risk-adjusted probability measure \mathbb{Q} :

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}} [\exp(-i_t) P_{t+1,n-1}] \quad (12)$$

By combining the dynamics of the pricing factors (9) and the short-term interest rate (6) with (12), the bond prices can be shown to be exponentially affine function in the pricing factors:

$$P_{t,n} = \exp(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t) \quad (13)$$

where the scalar $\mathcal{A}_n \equiv \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $\mathcal{B}_n \equiv \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1})$, a $1 \times K$ vector, are recursively defined loadings:²¹

$$\begin{aligned} \mathcal{A}_n &= -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}_{n-1}' + \mathcal{B}_{n-1} \boldsymbol{\mu}^{\mathbb{Q}} \\ \mathcal{B}_n &= -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1} \boldsymbol{\Phi}^{\mathbb{Q}} \end{aligned}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}'_K$ ensuring that the price of a ‘zero-period’ bond is one.

The continuously compounded yield on an n -day zero-coupon bond at time t , $y_{t,n} = -\frac{1}{n} \ln(P_{t,n})$, is given by:

$$y_{t,n} = A_n + B_n \mathbf{x}_t \quad (14)$$

where $A_n \equiv -\frac{1}{n} \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $B_n \equiv -\frac{1}{n} \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1})$.

The risk-neutral yield on an n -day bond reflects the expectation of the average short-term

²⁰See appendix C.2 for a formal derivation of these expressions.

²¹See appendix C.1 for a formal derivation of these expressions.

interest rate over the n -day life of the bond, corresponding to the yields that would prevail if investors were risk-neutral.²² That is, the yields that would arise under the expectations hypothesis of the yield curve. The risk-neutral yields can be calculated using:

$$\tilde{y}_{t,n} = \tilde{A}_n + \tilde{B}_n \mathbf{x}_t \quad (15)$$

where $\tilde{A}_n \equiv -\frac{1}{n} \mathcal{A}_n(\delta_0, \delta_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $\tilde{B}_n \equiv -\frac{1}{n} \mathcal{B}_n(\delta_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1})$.²³ Note that, the risk-neutral yields are attained, *inter alia*, using parameters specific to the actual probability measure \mathbb{P} , $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$. But, because no-arbitrage is assumed, the bonds are priced under the risk-adjusted measure \mathbb{Q} , so the fitted yields are attained, *inter alia*, by using parameters specific to the risk-adjusted probability measure \mathbb{Q} , $\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\}$.

The spot term premium on an n -day bond is defined as the difference between the fitted yield (14) and the risk-neutral yield (15):

$$tp_{t,n} = y_{t,n} - \tilde{y}_{t,n} \quad (16)$$

3.2 Unaugmented GADTSMs and the Identification Problem

Numerous studies have documented problems with separately identifying expectations of future short-term interest rates (risk-neutral yields) from term premia (e.g. Bauer et al., 2012; Kim and Orphanides, 2012; Guimarães, 2014). The underlying source of difficulty is an informational insufficiency, which gives rise to finite sample bias.

The unaugmented model uses zero-coupon bond yield data as its sole input. This data provides a complete set of information about the dynamic evolution of the cross-section of yields — the yield curve. This provides sufficient information to accurately identify the risk-adjusted \mathbb{Q} dynamics — specifically, the parameters $\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\}$ in (9) — which (14) shows are of direct relevance to estimating *actual* yields. However, if there is no additional information and the sample of yields contains too few interest rate cycles,²⁴ this data is not sufficient for the identification of the actual \mathbb{P} dynamics — specifically, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ in (5) — which (15) illustrates are of relevance to the estimation of *risk-neutral* yields.²⁵ Estimates of $\boldsymbol{\Phi}$ for the autoregressive process in (5) will suffer from finite sample bias. In particular, the persistent yields will have persistent pricing factors, so maximum likelihood or ordinary least squares estimates of the persistence parameters of the vector autoregressive process in (5) $\boldsymbol{\Phi}$ will be biased downwards.²⁶ That is, the estimated $\hat{\boldsymbol{\Phi}}$ will understate the true persistence of the pricing factors, implying a spuriously fast mean reversion of future short-term interest rates. Because,

²²There is a small difference between risk-neutral yields and expected yields due to a convexity effect. In the homoskedastic model considered here, these effects are constant for each maturity and, in practice, small, corresponding to the $\frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}_{n-1}'$ term in the recursive expression for \mathcal{B}_n above.

²³See appendix C.3 for a formal derivation of these expressions.

²⁴Kim and Orphanides (2012, p. 242) state that 5 to 15-year samples may contain too few interest rate cycles.

²⁵Note that because $\boldsymbol{\mu} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Sigma} \boldsymbol{\lambda}_0$ and $\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\mathbb{Q}} + \boldsymbol{\Sigma} \boldsymbol{\Lambda}_1$, estimates of the time-varying market prices of risk, $\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1$, are required to estimate $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ and the risk-neutral yields.

²⁶This is a multivariate generalisation of the downward bias in the estimation of autoregressive parameters by ordinary least squares in the univariate case.

in the model, agents form expectations of future short-term interest rates based on estimates of pricing factor mean reversion in $\widehat{\Phi}$, their estimates of the future short-term interest rate path will mean revert spuriously quickly too. Consequently, the estimated risk-neutral yields, which summarise the average of the expected path of future short-term interest rates, will vary little and will not accurately reflect the evolution of interest rate expectations.

The magnitude of the finite sample bias is increasing in the persistence of the data. So for daily frequency yield data, which is highly persistent, the bias will be more severe. This not only motivates the augmentation of the GADTSM with additional data, but motivates the use of additional *daily frequency* data, namely OIS rates.

4 The OIS-Augmented Model

I estimate the OIS-augmented model using Kalman filter-based maximum likelihood. The Kalman filtering approach is particularly convenient for the augmentation of GADTSMs, as it can handle mixed-frequency data. Specifically, for OIS-augmentation, this allows estimation of the GADTSM for periods extending beyond that for which OIS rates are available.²⁷

To implement the Kalman filter-based estimation, I use (5), the vector autoregression for the latent pricing factors under the actual probability measure \mathbb{P} , as the transition equation.

The observation equation depends on whether OIS rates are observed on day t or not. On days when the OIS rates are *not* observed (i.e. days prior to January 2002), the observation equation is formed by stacking the N yield maturities in (14) to form:

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\mathbf{x}_t + \boldsymbol{\Sigma}_Y \mathbf{u}_t \quad (17)$$

where: $\mathbf{y}_t = [y_{t,n_1}, \dots, y_{t,n_N}]'$ is the $N \times 1$ vector of bond yields; $\mathbf{A} = [A_{n_1}, \dots, A_{n_N}]'$ is an $N \times 1$ vector and $\mathbf{B} = [B'_{n_1}, \dots, B'_{n_N}]'$ is an $N \times K$ matrix of bond-specific loadings; $A_{n_\iota} = -\frac{1}{n_\iota} \mathcal{A}_{n_\iota}(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n_\iota-1}, \mathcal{B}_{n_\iota-1})$ and $B_{n_\iota} = -\frac{1}{n_\iota} \mathcal{B}_{n_\iota}(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n_\iota-1})$ are the bond-specific loadings; and $\iota = 1, 2, \dots, N$ such that n_ι denotes the maturity of bond ι in days. The $N \times 1$ vector $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}_N, \mathbf{I}_N)$ denotes the yield measurement error, where $\mathbf{0}_N$ is an N -vector of zeros and \mathbf{I}_N is an $N \times N$ identity matrix. Here, like much of the existing literature,²⁸ I impose a homoskedastic form for the yield measurement error, such that $\boldsymbol{\Sigma}_Y$ is an $N \times N$ diagonal matrix with common diagonal element σ_e , the standard deviation of the yield measurement error. The homoskedastic error is characterised by a single parameter σ_e , maintaining computational feasibility for an already high-dimensional maximum likelihood routine.

On days when OIS rates are observed, the Kalman filter observation equation is augmented with OIS rates. The following proposition illustrates that OIS rates can (approximately) be written as an affine function of the pricing factors with loadings A_j^{ois} and B_j^{ois} for J different OIS maturities, where $j = j_1, j_2, \dots, j_J$ denote the J OIS horizons in days. The loadings presented

²⁷This paper uses daily US OIS rates from 2002, the first date for which these rates are consistently available at all the relevant tenors on Bloomberg. Models are estimated from this date to directly isolate the effect of OIS rates on GADTSMs. However, given the Kalman filter method, the model can be estimated over longer periods.

²⁸See, for example, [Guimarães \(2014\)](#).

in this proposition are calculated by assuming that the expectations hypothesis (3) holds for the OIS tenors included in the model, an assumption that was verified in section 2 for the maturities used here. Moreover, the loadings explicitly account for the payoff structure of an OIS contract. It is in this respect that the technical setup of the OIS-augmented GADTSM most clearly differs from the survey-augmented model.

Proposition The j -day OIS rate on date t $i_{t,t+j}^{ois}$, where $j = j_1, j_2, \dots, j_J$, can be (approximately) written as an affine function of the pricing factors \mathbf{x}_t :

$$i_{t,t+j}^{ois} = A_j^{ois} + B_j^{ois} \mathbf{x}_t \quad (18)$$

where $A_j^{ois} \equiv \frac{1}{j} \mathcal{A}_j^{ois} \left(\delta_0, \delta_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{j-1}^{ois}, \mathcal{B}_{j-1}^{ois} \right)$ and $B_j^{ois} \equiv \frac{1}{j} \mathcal{B}_j^{ois} \left(\delta_1, \boldsymbol{\Phi}; \mathcal{B}_{j-1}^{ois} \right)$ are recursively defined as:

$$\begin{aligned} \mathcal{A}_j^{ois} &= \delta_0 + \delta_1' \boldsymbol{\mu} + \mathcal{A}_{j-1}^{ois} + \mathcal{B}_{j-1}^{ois} \boldsymbol{\mu} \\ \mathcal{B}_j^{ois} &= \delta_1' \boldsymbol{\Phi} + \mathcal{B}_{j-1}^{ois} \boldsymbol{\Phi} \end{aligned}$$

where $\mathcal{A}_0^{ois} = 0$ and $\mathcal{B}_0^{ois} = \mathbf{0}'_K$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

Proof: See appendix D.

Given this, the Kalman filter observation equation on the days OIS rates are observed is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{ois} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{ois} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{ois} \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \boldsymbol{\Sigma}_Y & \mathbf{0}_{N \times J} \\ \mathbf{0}_{J \times N} & \boldsymbol{\Sigma}_O \end{bmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^{ois} \end{bmatrix} \quad (19)$$

where, in addition to the definitions of \mathbf{y}_t , \mathbf{A} , \mathbf{B} , $\boldsymbol{\Sigma}_Y$ and \mathbf{u}_t above, $\mathbf{i}_t^{ois} = [i_{t,j_1}^{ois}, \dots, i_{t,j_J}^{ois}]'$ is the $J \times 1$ vector of OIS rates; $\mathbf{A}^{ois} = [A_{j_1}^{ois}, \dots, A_{j_J}^{ois}]'$ is a $J \times 1$ vector and $\mathbf{B}^{ois} = [B_{j_1}^{ois}, \dots, B_{j_J}^{ois}]'$ is a $J \times K$ matrix of OIS-specific loadings; $\mathbf{0}_{N \times J}$ and $\mathbf{0}_{J \times N}$ denote $N \times J$ and $J \times N$ matrices of zeros respectively; and $\mathbf{u}_t^{ois} \sim \mathcal{N}(\mathbf{0}_J, \mathbf{I}_J)$ denotes the OIS measurement error, where $\mathbf{0}_J$ is an J -vector of zeros and \mathbf{I}_J is an $J \times J$ identity matrix. The inclusion of the measurement error permits non-zero OIS forecast errors, imposing that the forecast error is zero *on average*. I compared two parameterisations of $\boldsymbol{\Sigma}_O$; a homoskedastic model, with common diagonal elements in $\boldsymbol{\Sigma}_O$, and a heteroskedastic model, with distinct diagonal elements. A likelihood ratio test of the two did not reject the null hypothesis that all diagonal elements are equal, so I impose a homoskedastic form for the OIS measurement error such that $\boldsymbol{\Sigma}_O$ has common diagonal element σ_o , the standard deviation of the OIS measurement error, and zero elsewhere. The homoskedastic OIS measurement errors also provide computational benefits, as there are fewer parameters to estimate than if a more general covariance structure was permitted.²⁹

²⁹Kim and Orphanides (2012) and Guimarães (2014) impose homoskedasticity on the survey measurement errors in their Kalman filter setup for this reason.

5 Methodology

To compare the OIS-augmented model with the existing literature, I estimate the following GADTSM-variants: (i) an unaugmented OLS/ML model, estimated using the [Joslin et al. \(2011\)](#) identification scheme, where K portfolios of yields are observed without error and are measured with the first K estimated principal components of the bond yields; (ii) the [Bauer et al. \(2012\)](#) bias-corrected model; (iii) a survey-augmented model, using expectations of future short-term interest rates for the subsequent four quarters as an additional input, estimated with the Kalman filter using the algorithm of [Guimarães \(2014\)](#) (see appendix E for details);³⁰ and (iv) the OIS-augmented model.

5.1 Data

In all models, bond yields \mathbf{y}_t of the following maturities are used: 3 and 6 months, 1 year, 18 months, 2 years, 30 months, 3 years, 42 months, 4 years, 54 months, 5, 7 and 10 years.³¹ For the 3 and 6-month yields, I use US T-Bill rates in accordance with much of the existing dynamic term structure literature and evidence from [Greenwood, Hanson, and Stein \(2015\)](#), who document a marked wedge between 1-26-week T-Bill rates and corresponding maturity fitted zero-coupon bond yields.³² The remaining rates are from the continuously compounded zero-coupon yields of [Gürkaynak, Sack, and Wright \(2007a\)](#). This data is constructed from daily-frequency fitted Nelson-Siegel-Svensson yield curves. Using the parameters of these curves, which are published along with the estimated zero-coupon yield curve, I back out the cross-section of yields for the 11 maturities from 1 to 10-years.³³

OIS rates are from *Bloomberg*. I use combinations of 3, 6, 12 and 24-month OIS rates in the OIS-augmented models. The choice of these maturities is motivated by evidence in section 2 and [Lloyd \(2017b\)](#). I estimate three variants of the OIS-augmented model. The first, baseline setup, includes the 3, 6, 12 and 24-month OIS rates (4-OIS-Augmented model). The second and third models include the 3, 6 and 12-month (3-OIS-Augmented model) and 3 and 6-month (2-OIS-Augmented model) tenors respectively.³⁴ Of the three OIS-augmented models, I find that the 4-OIS-Augmented model provides risk-neutral yields that best fit the evolution of interest rate expectations, in and out-of-sample.

Since US OIS rates are consistently available from January 2002, the baseline sample period runs from January 2002 to December 2016 to isolate the effect of OIS augmentation.

³⁰For direct comparison to my OIS-augmented model, I estimate the survey-augmented model by applying the algorithm of [Guimarães \(2014\)](#), who also uses the same [Joslin et al. \(2011\)](#) identification scheme. [Kim and Orphanides \(2012\)](#) implement a different identification scheme in the estimation of their survey-augmented model. Like [Guimarães \(2014\)](#), I use survey expectations from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia, including forecasts of the 3-month T-Bill rate for the remainder of the current quarter and the first, second, third and fourth quarters ahead.

³¹These yield maturities correspond to those used by [Adrian, Crump, and Moench \(2013\)](#).

³²The T-Bill rates are converted from their discount basis to the yield basis.

³³The Nelson-Siegel-Svensson yield curve used to back out the cross-section of yields at a daily frequency is reported in equation (22) of [Gürkaynak et al. \(2006\)](#), an earlier working paper version of [Gürkaynak et al. \(2007a\)](#).

³⁴Because the results from the 2-OIS-augmented model are inferior to those from the 4 and 3-OIS-augmented models, I present results for the 2-OIS-augmented model in appendix F.

In accordance with the well-rehearsed evidence of [Litterman and Scheinkman \(1991\)](#), that the first three principal components of bond yields explain well over 95% of their variation, I estimate the models with three pricing factors ($K = 3$).³⁵ By using the three-factor specification, for which the pricing factors have a well-understood economic meaning (the level, slope and curvature of the yield curve respectively), I am able to isolate and explain the economic mechanisms through which the OIS-augmented model provides superior estimates of expectations of future short-term interest rates *vis-à-vis* the unaugmented, bias-corrected and survey-augmented models.

5.2 Estimation

The OIS-augmented model relies on Kalman filter-based maximum likelihood estimation, for which the pricing factors \mathbf{x}_t are latent. Normalisation restrictions must be imposed on the parameters to achieve identification. For this, I appeal to the normalisation scheme of [Joslin et al. \(2011\)](#), which “allows for computationally efficient estimation of G[A]DTSMs” ([Joslin et al., 2011](#), p. 928) and fosters faster convergence to the global optimum of the model’s likelihood function than other normalisation schemes (e.g. [Dai and Singleton, 2000](#)).³⁶ This permits a two-stage approach to estimating the OIS-augmented model.

To benefit fully from the computational efficiency of the [Joslin et al. \(2011\)](#) normalisation scheme, I first estimate the unaugmented GADTSM (hereafter, labelled the OLS/ML model), presented in section 3.1, assuming that K portfolios of yields are priced without error, to attain initial values for the Kalman filter used in the second estimation stage. In particular, these K yield ‘portfolios’, \mathbf{x}_t , correspond to the first K estimated principal components of the bond yields. Under the [Joslin et al. \(2011\)](#) normalisation, this itself enables a two sub-stage estimation: first the \mathbb{P} parameters are estimated by OLS on equation (5) using the K estimated principal components in the vector \mathbf{x}_t ; second the \mathbb{Q} parameters are estimated by maximum likelihood (see appendix E for details).

Having attained these OLS/ML parameter estimates, I subsequently estimate the OIS-augmented model — which assumes all yields are observed with error — using the OLS/ML parameter estimates as initial values for the Kalman filter-based maximum likelihood routine.

³⁵I also estimate a four-factor specification in the light of evidence by [Cochrane and Piazzesi \(2005, 2008\)](#) and [Duffee \(2011\)](#) who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields. These results are reported in appendix F.2.

³⁶The computational benefits of the [Joslin et al. \(2011\)](#) normalisation scheme arise because it only imposes restrictions on the short-term interest rate i_t and the factors \mathbf{x}_t under the \mathbb{Q} probability measure. Consequently, the \mathbb{P} and \mathbb{Q} dynamics of the model do not exhibit strong dependence. Under the [Dai and Singleton \(2000\)](#) scheme, restrictions on the volatility matrix Σ , which influences both the \mathbb{P} and \mathbb{Q} evolution of the factors (see equations (5) and (9)), create a strong dependence between the parameters under the two probability measures, engendering greater computational complexity in the estimation.

6 Term Structure Results

6.1 Model Fit

This sub-section discusses four aspects of model fit: estimated bond yields, estimated OIS rates, estimated pricing factors and parameter estimates.

6.1.1 Fitted Bond Yields

Importantly, augmentation of the GADTSM with OIS rates does not compromise the overall fit of the model with respect to actual bond yields. The fit of actual yields is strikingly similar across all the models. Figure 2 illustrates that the residuals of the 2-year fitted yield from the OLS/ML, bias-corrected, survey-augmented, 4 and 3-OIS-augmented models follow similar qualitative and quantitative paths.³⁷

The similar fit of actual yields is intuitive. I augment the GADTSM with OIS rates to provide additional information with which to better estimate parameters under the actual probability measure $\mathbb{P} \{ \boldsymbol{\mu}, \boldsymbol{\Phi} \}$, which directly influence estimates of the risk-neutral yields. Estimates of the fitted yield depend upon the risk-adjusted measure \mathbb{Q} parameters $\{ \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}} \}$, which are not directly influenced by the OIS rates in the model, and are well-identified with bond yield data that provide information on the dynamic evolution of the cross-section of yields.

6.1.2 Fitted OIS Rates

Alongside estimates of the actual bond yield, the OIS-augmented models also provide fitted values for OIS rates. Figure 3 plots the 3, 6, 12 and 24-month OIS rates alongside the corresponding-maturity fitted-OIS rates from the 4, 3 and 2-OIS-augmented models.

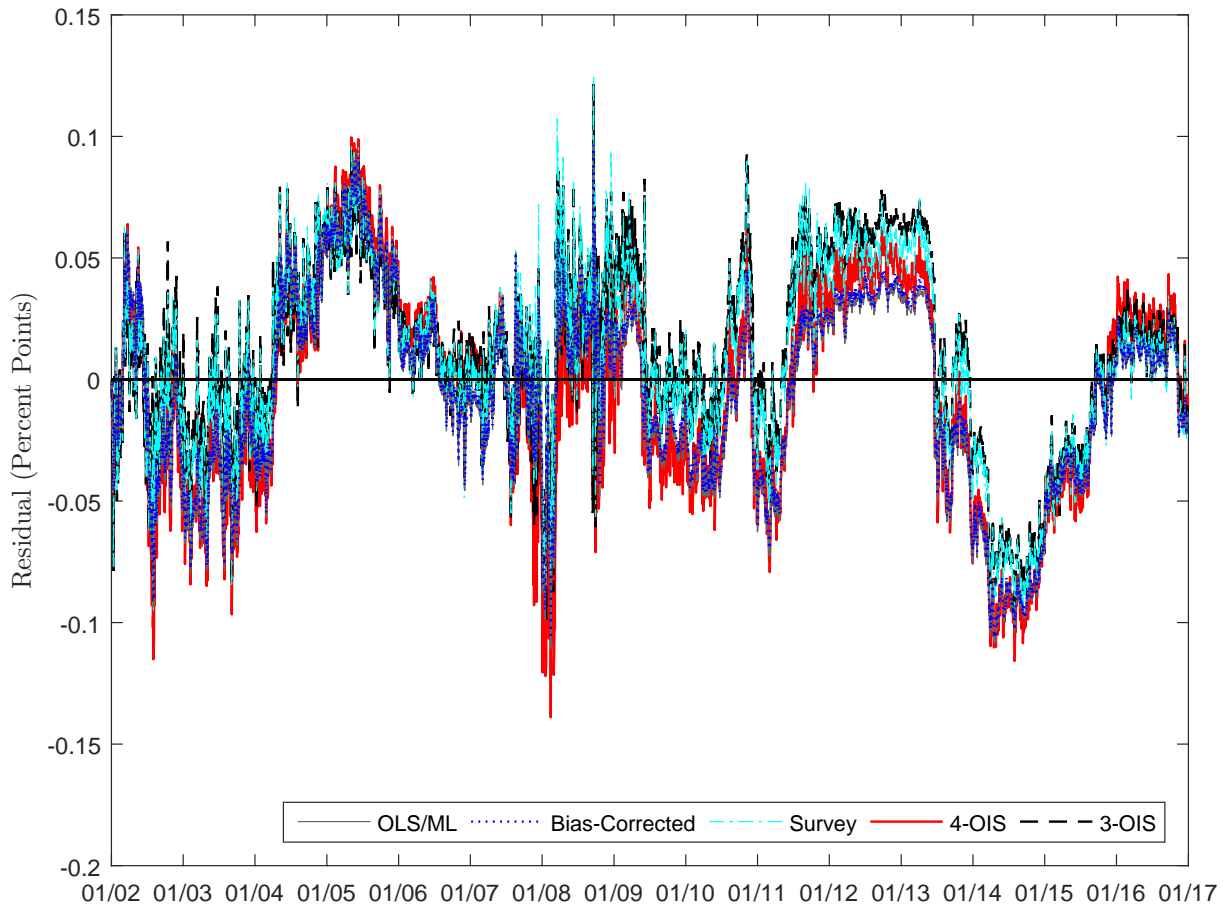
The plots illustrate that the OIS-augmented models provide accurate estimates of actual OIS rates.³⁸ The 4-OIS-augmented model provides the best fit for the 6, 12 and 24-month OIS rates, while the 2-OIS-augmented model best fits the 3-month OIS rate. Although the differences between the OIS-augmented models at the 3-month horizon are marginal, the 4-OIS-augmented model fits the 24-month OIS rate substantially better than the 3 and 2-OIS-augmented models. This is unsurprising, as this OIS tenor is observed in the 4-OIS-augmented model. The 2-OIS-augmented model fits the 1 and 2-year OIS rates least well. This is unsurprising, as it uses the fewest OIS rates as observable inputs.

The fact the OIS-augmented models do not fit OIS rates as well as they fit bond yields — the quantitative value of OIS-RMSE (approximately 10 basis points) is almost double that of the bond yield-RMSE (approximately 5 basis points) — is neither worrying nor surprising. The GADTSM uses thirteen bond yields as inputs to estimate the cross-section of fitted yields in every time period, whereas only four OIS rates are used to fit the cross-section of OIS rates. Moreover, adding additional OIS rates is not warranted given that they are included to improve

³⁷Table 7, in appendix F.1.1, provides more detailed evidence of the similar actual yield fit of the models, documenting the root mean square error (RMSE) for each model at each yield maturity. Over the 13 maturities, the average RMSE of each model is around 5 basis points.

³⁸Table 8, in appendix F.1.2, provides more detailed numerical evidence on this.

Figure 2: Residual of the 2-Year Fitted Yield from GADTSMs



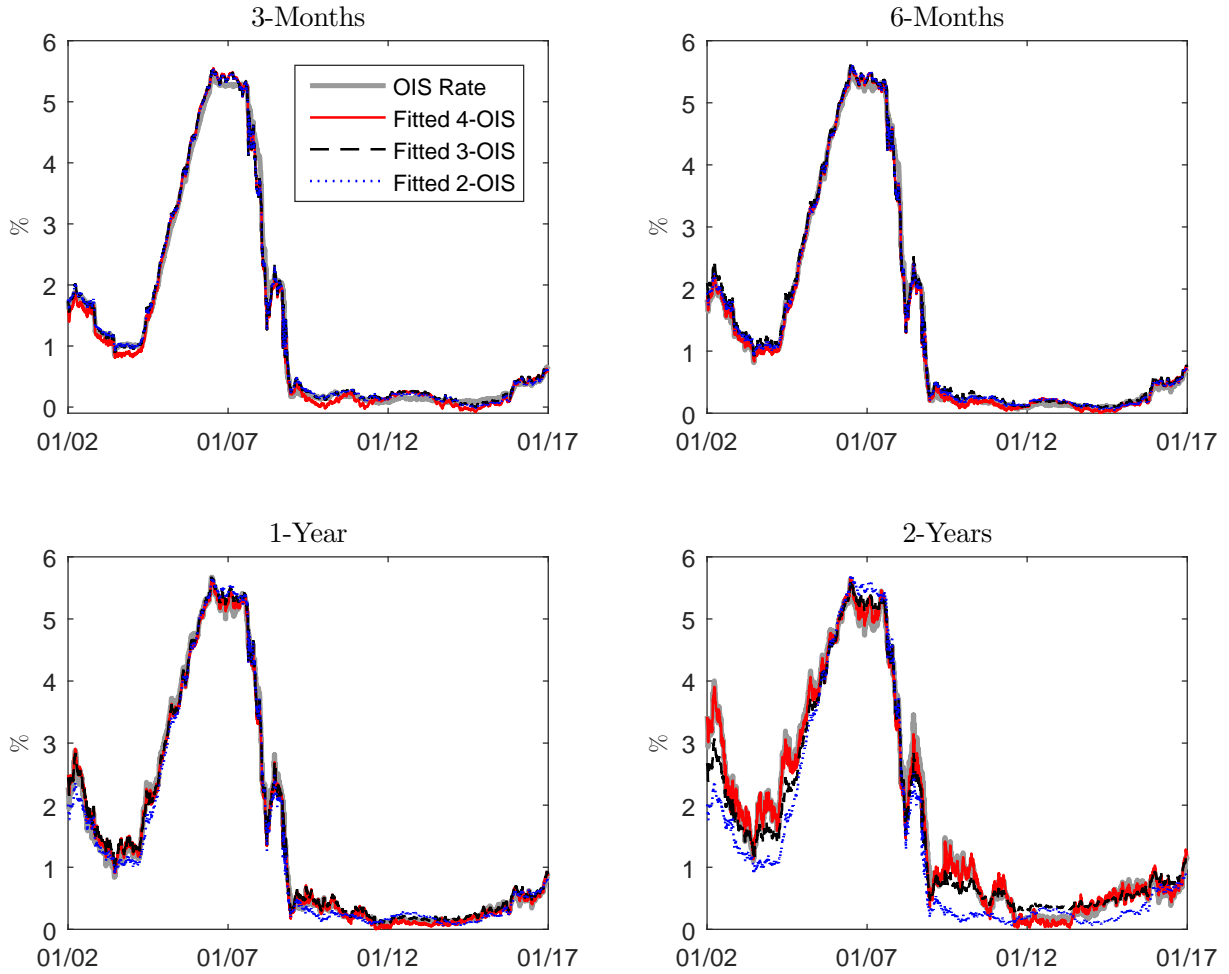
Note: Residuals of the 2-year fitted yield from five GADTSMs: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

the fit of model-implied interest rate expectations and that longer-maturity OIS rates contain significant term premia (Lloyd, 2017b).

6.1.3 Pricing Factors

Of additional interest for the OIS-augmented model is whether the inclusion of OIS rates affects the model’s pricing factors \mathbf{x}_t . To investigate this, I compare the estimated principal components of the bond yields — used as pricing factors in the OLS/ML model — to the estimated model-implied pricing factors from Kalman filter estimation of the OIS-augmented models. Figure 4 plots the time series of the first three principal components, estimated from the panel of bond yields, and the estimated pricing factors from the 4-OIS-augmented model. For all three factors, the Kalman filter-implied pricing factors are nearly identical to the estimated principal

Figure 3: Fitted OIS Rates from the OIS-Augmented Models



Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with three pricing factors using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

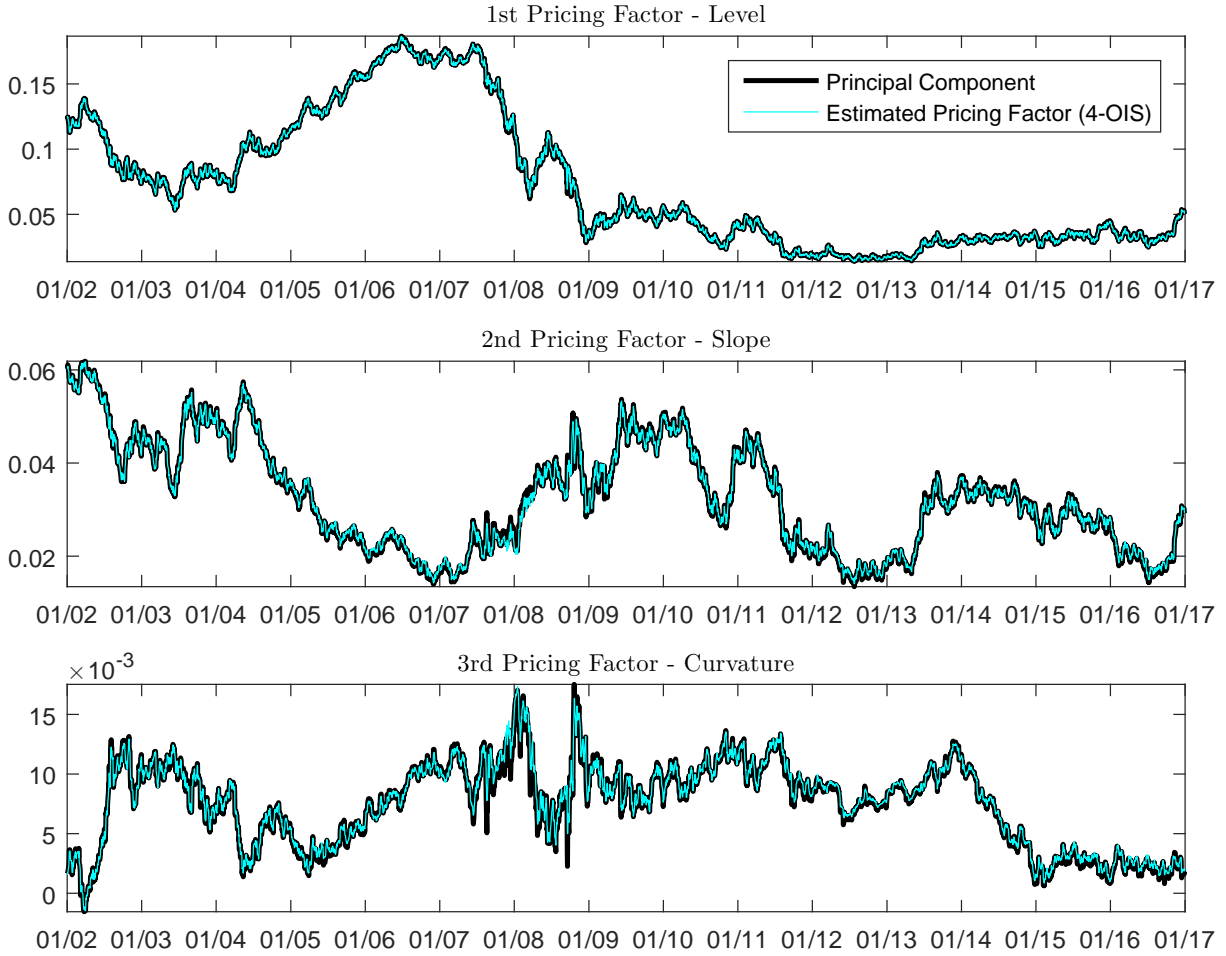
components.³⁹ This implies that OIS rates do not include any additional information, over and above that in bond yields, of value in fitting the actual yields. This, again, is intuitive: OIS rates are included in the GADTSM to provide information useful for the identification of the risk-neutral yields, not the fitted yields.

6.1.4 Parameter Estimates

Recall, from section 3.2, that informational insufficiency in GADTSMs gives rise to finite sample bias. Persistent yields will have persistent pricing factors, resulting in estimates of the persistence parameters $\hat{\Phi}$ that are biased downwards. Following Bauer et al. (2012), I numerically assess the extent to which OIS-augmentation reduces finite sample bias by reporting the maximum eigenvalues of the estimated persistence parameters $\hat{\Phi}$. The higher the maximum

³⁹Table 9, in appendix F.1.3, demonstrates that the summary statistics of the estimated principal components and pricing factors are very similar too.

Figure 4: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model, implied by the Kalman filter.

eigenvalue, the more persistent the estimated process.

As a benchmark, the maximum absolute eigenvalue of $\hat{\Phi}$ for the unaugmented OLS/ML model is 0.9985. The maximum absolute eigenvalue of $\hat{\Phi}$ for the survey-augmented model is 0.9983. However, for the 4-OIS-augmented model, the corresponding figure is 0.9988, indicating that, in comparison to the unaugmented model, augmentation with OIS rates does serve to mitigate finite sample bias.⁴⁰ This indicates that OIS-augmentation does help to resolve the informational insufficiency in GADTSMs, and its associated symptoms. However, to assess this more thoroughly, a comparison of model-implied interest rate expectations is necessary. A well-identified model should accurately reflect the evolution of interest rate expectations.

Appendix F.1.4 presents further evidence to indicate that the OIS-augmentation resolves the identification problem, by studying the stability of interest rate expectation estimates using

⁴⁰The corresponding statistic for the bias-corrected model, which performs bias-correction directly on the estimated Φ , is 1.0000 (to four decimal places). However, the ‘true’ pricing factor persistence is unknown.

different sample periods of data. Estimates of interest rate expectations (on a given date) from the unaugmented OLS/ML model are more liable to vary as the sample period changes. In contrast, estimates from the 4-OIS-augmented model are remarkably stable, indicating that OIS-augmentation does provide useful information for the identification of interest rate expectations, which improves the model’s usefulness for real-time policy analysis.

6.2 Model-Implied Interest Rate Expectations

The central focus of this paper is the identification and estimation of interest rate expectations within GADTSMs. Panels A and B of figure 5 plot the 2-year risk-neutral yields and term premia from the GADTSMs estimated between January 2002 and December 2016, respectively.

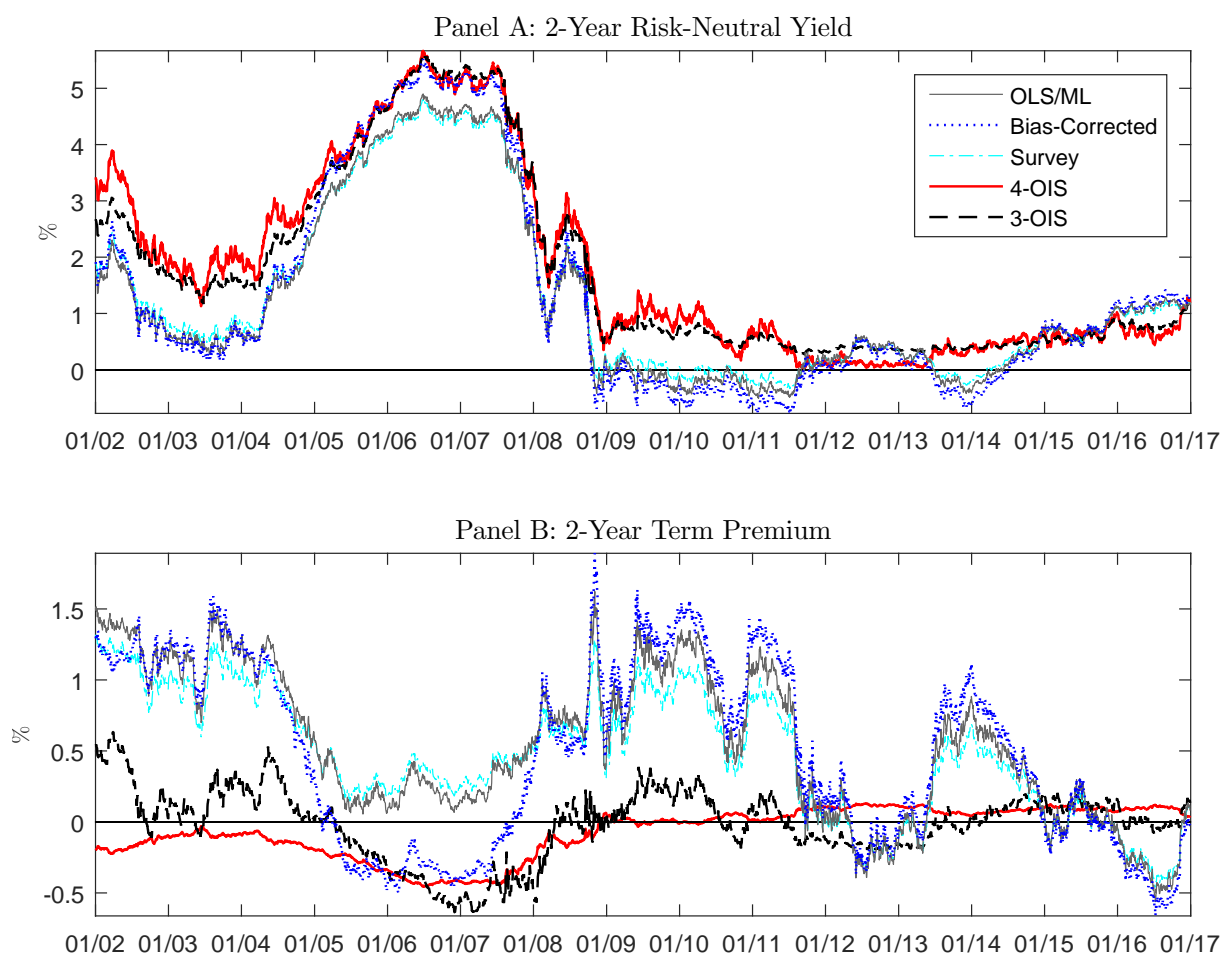
Panel A offers illustrative evidence of the effect of OIS-augmentation on the GADTSM estimates of expected future short-term interest rates. Over the 2002-2016 sample, the five models exhibit similar qualitative patterns, rising to peaks and falling to troughs at similar times. However, there are a number of notable differences between the series that help to illustrate the benefits of OIS-augmentation.

For the majority of the 2002-2016 sample, the OIS-augmented models generate 2-year risk-neutral yields that exceed those from the OLS/ML and bias-corrected models.⁴¹ Moreover, marked differences exist in the evolution of risk-neutral yield estimates from the models from late-2008 onwards. These differences have counterfactual implications for the efficacy of monetary policy at this time. First, from late-2008 to late-2011, the risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying counterfactual expectations of negative interest rates. In contrast, unlike the other models, the risk-neutral yields implied by the 3 and 4-OIS-augmented models obey a zero lower bound, with estimated interest rate expectations never falling negative, despite the fact that additional restrictions are not imposed to achieve this. This is true at all horizons, and represents an important contribution in the light of recent computationally burdensome proposals for term structure modelling at the zero lower bound (e.g. [Christensen and Rudebusch, 2013a,b](#)).

Second, between mid-2011 and 2013, the 2-year risk neutral yields from the OLS/ML and bias-corrected models reach a peak, indicating an increase in expected future short-term interest rates over a 2-year horizon. In contrast, during the same period, the 2-year risk-neutral yield estimates from the 3-OIS-augmented model remain broadly stable, while the corresponding estimates from the 4-OIS-augmented model fall slightly to a trough. From August 2011, the Federal Reserve engaged in calendar-based forward guidance designed to influence investors’ expectations of future short-term interest rates, signalling that interest rates would be kept at a low level for an extended period of time. For instance, on August 9, 2011, the Federal Open Market Committee (FOMC) stated that it expected to keep the federal funds rate near zero “at least through mid-2013.” This, and other forward guidance statements, were effective at

⁴¹Longer-horizon (i.e. 10-year) risk-neutral yields from the OIS-augmented models also exceed those from the OLS/ML and bias-corrected models. This is consistent with [Meldrum and Roberts-Sklar \(2015\)](#), who argue that unaugmented models provide “implausibly low estimates of long-term expected future short-term interest rates” (p. 1), “which in turn means that long-maturity term premium estimates are likely to be too high” (p. 3).

Figure 5: Estimated Yield Curve Decomposition



Note: Estimated risk-neutral yields (panel A) and term premia (panel B) from each of five GADTSMs, respectively. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

deferring investors' expectations of future rate rises between mid-2011 and 2013. [Swanson and Williams \(2014\)](#) show that private sector expectations of the time until a US rate rise, from Blue Chip surveys, jumped from between 2 and 5 quarters to 7 or more quarters. In this respect, the finding that expectations of future short-term interest rates increased during this period, as implied by the OLS/ML and bias-corrected models, wrongly suggests that forward guidance policy was counterproductive. These models predict that investors began to expect rate rises sooner rather than later. The OIS-augmented models imply that investors were expecting rate rises no sooner, and possibly slightly later, than they had in previous period. Subsequent quantitative analysis further demonstrates that the OIS-augmented models provide superior estimates of interest rate expectations during this period.

In figure 5, the estimated 2-year term premium from the 4-OIS-augmented model is persistently negative from 2002 to 2008. This is a direct consequence of the accurate fitting of

risk-neutral yields. However, this feature is not true for all maturities; the estimated term premia at longer-horizons are frequently and persistently positive. For instance, the 10-year term premium from the 4-OIS-augmented model peaks at 79 basis points in late-2008.

6.2.1 Risk-Neutral Yields and Federal Funds Futures

To quantitatively evaluate the GADTSM-implied risk-neutral yields, I first compare them to federal funds futures rates. A federal funds futures contract pays out at maturity based on the average effective federal funds rate realised during the calendar month specified in the contract. Federal funds futures rates have long been used as measures of investors' expectations of future short-term interest rates (Lao and Mirza, 2015) and many authors have assessed the quantitative accuracy of federal funds futures rates as predictors of future monetary policy. Gürkaynak et al. (2007b) conclude that, in comparison to a range of other financial market-based measures of interest rate expectations (not including OIS rates), federal funds futures rates provide the superior forecasts of future monetary policy out to 6 months. Lloyd (2017b) finds that, at a monthly frequency between 2002 and 2016, the average *ex post* realised excess returns on 1 to 11-month federal funds futures contracts are insignificantly different from zero at the 5% significance level. Motivated by this evidence, I compare estimated risk-neutral yields from each of the GADTSMs to expectations implied by 1 to 11-month federal funds futures contracts with matching horizon.

To facilitate this comparison, I first calculate 1, 2, ..., 11-month risk-neutral yields using the estimated model parameters from each GADTSM. I then calculate risk-neutral 1-month *forward* yields using the estimated risk-neutral yields.⁴² Like federal funds futures contracts, the risk-neutral 1-month forward rates settle based on outcomes during a 1-month period in the future. However, because of the settlement structure of federal funds futures contracts, I only compare risk-neutral forward yields and federal funds futures rates on the final day of each calendar month.⁴³ I find that the risk-neutral forward yields from the OIS-augmented models most closely align with the expectations implied by federal funds futures rates with matching horizon, implying that they provide superior estimates of investors' expected future short-term interest rates compared to other GADTSMs.

Table 1 provides formal evidence in support of this conclusion, presenting the RMSE of risk-neutral 1-month forward yields from different GADTSMs and corresponding-horizon federal funds futures rates. On a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates, as measured by federal funds futures

⁴²To calculate the risk-neutral forward rate \tilde{f}_{t_1, t_2} from day t_1 to day t_2 , I use the following formula:

$$(1 + \tilde{y}_2)^{d_2} = (1 + \tilde{y}_1)^{d_1} (1 + \tilde{f}_{t_1, t_2})^{d_2 - d_1}$$

where \tilde{y}_1 (\tilde{y}_2) is the risk-neutral yield for the time period $(0, t_1)$ ($(0, t_2)$) and d_1 (d_2) is the length of time between time 0 and time t_1 (t_2) in years.

⁴³See Lloyd (2017b) for a detailed description of the settlement structure of federal funds futures contracts. The salient point is that an n -month federal funds futures contract traded on day t_j of the calendar month t has the same settlement period as an n -month contract traded on a different day t_k in the same calendar month t . For this reason, the horizon of a federal funds futures contract and the risk-neutral forward yield only align on the final calendar day of each month.

Table 1: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the Risk-Neutral 1-Month Forward Yields *vis-à-vis* Corresponding-Horizon Federal Funds Futures Rates

Sample: January 2002 to December 2016					
Horizon	OLS/ML	BC	Survey	3-OIS	4-OIS
0 to 1 Months	0.2230	0.2196	0.2025	0.1649	0.1823
1 to 2 Months	0.2134	0.2049	0.1802	0.1201	0.1293
2 to 3 Months	0.2307	0.2167	0.1842	0.1205	0.0929
3 to 4 Months	0.2678	0.2496	0.2147	0.1394	0.0828
4 to 5 Months	0.3099	0.2888	0.2534	0.1552	0.0898
5 to 6 Months	0.3573	0.3344	0.2981	0.1593	0.1021
6 to 7 Months	0.4103	0.3865	0.3480	0.1576	0.1176
7 to 8 Months	0.4648	0.4422	0.3983	0.1545	0.1316
8 to 9 Months	0.5243	0.5030	0.4528	0.1530	0.1429
9 to 10 Months	0.9773	0.9599	0.9200	0.6755	0.6564
10 to 11 Months	1.3392	1.3297	1.2776	0.9915	0.9635

Note: RMSE of the risk-neutral 1-month forward yields from each of the five GADTSMs in comparison to corresponding-horizon federal funds futures rates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The risk-neutral forward yields and the federal funds futures rates are compared on the final day of each calendar month. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

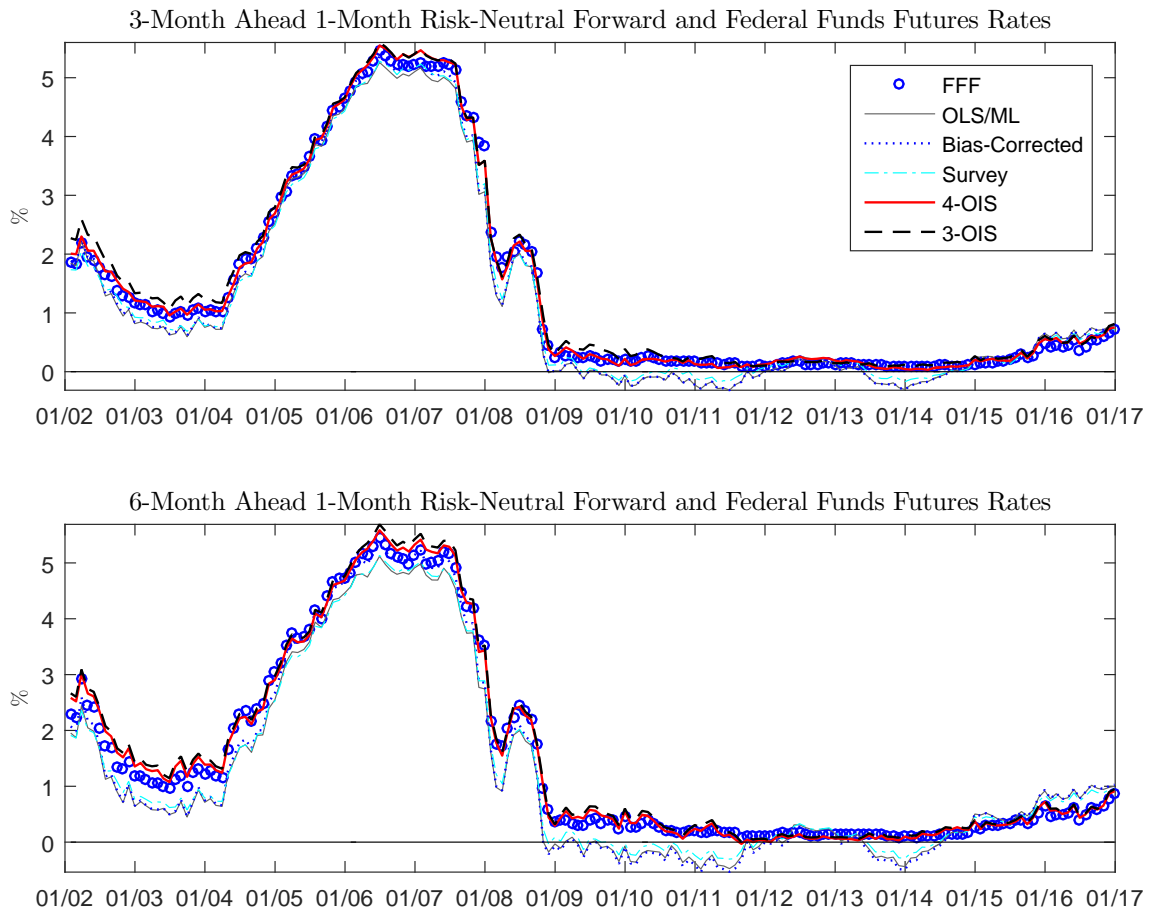
rates, at every horizon. The 4-OIS-augmented model outperforms the 3-OIS-augmented model at all but two horizons. Even at extremely short horizons, the benefits of OIS-augmentation are large: the RMSE fit of the unaugmented OLS/ML and bias-corrected models at the 3 to 4 month horizon is over three times larger than that of the 4-OIS-augmented model.

Despite fitting federal funds futures-implied interest rate expectations worse than the OIS-augmented models, the survey-augmented model does perform better than the unaugmented OLS/ML and bias-corrected models in this regard. This supports the claim that, while survey-augmentation does help to reduce the informational insufficiency problem in GADTSMs, quarterly frequency survey expectations are not sufficient for the accurate identification of interest rate expectations at higher frequencies.

Figure 6 provides visual comparison of the risk-neutral 1-month forward yields and federal funds futures rates at the 3 to 4 and 6 to 7 month horizons. Here, I plot the risk-neutral 1-month forward yields from the unaugmented OLS/ML, bias-corrected, survey-augmented, 4-OIS-augmented and 3-OIS-augmented GADTSMs, as well as corresponding-horizon federal funds futures rates. The plot highlights the causes of the differences in fit highlighted by table 1. Two important observations follow.

First, in the pre-crisis period (to mid-2007), the OLS/ML and survey-augmented GADTSMs generate estimated risk-neutral forward yields that persistently fall below the corresponding-horizon federal funds futures rates. The bias-corrected risk-neutral forward yields also fall below the corresponding-horizon federal funds futures rates until 2005. In contrast, the estimated risk-

Figure 6: Estimated Risk-Neutral 1-Month Forward Yields and Comparable-Horizon Federal Funds Futures (FFF) Rates



Note: Estimated 3 month (top panel) and 6 month (bottom panel) ahead 1-month risk-neutral forward yields from each of five GADTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates, plotted on the final day of each calendar month. All figures are in annualised percentage points.

neutral forward yields from the OIS-augmented models align more closely with federal funds futures rates during this period, especially at the 3 to 4-month horizon.

Second, from late-2008 until late-2011, and from mid-2013 to mid-2014, the risk-neutral forward yields from the OLS/ML, bias-corrected, and survey-augmented models differ greatly from the corresponding-horizon federal funds futures rates. Moreover, these models offer counterfactual predictions for the evolution of interest rate expectations during this period. In particular, in these periods, the risk-neutral forward yields from the OLS/ML, bias-corrected and survey-augmented models are persistently negative, implying that investors expected future short-term interest rates to fall negative. Moreover, from late-2011 to mid-2012, the risk-neutral forward yields from the OLS/ML and bias-corrected models rise to a peak. Not only is this contrary to the policy narrative at the time — policymakers were engaging in calendar-based forward

guidance that sought to push back the date investors expected policy rates to lift-off from their lower bound — it is also counterfactual with respect to market-implied interest rate expectations. In contrast, the OIS-augmented models align closely with federal funds futures-implied expectations for most of the 2009-2016 period.

Overall, the comparison of risk-neutral forward yields and federal funds rates supports the claim that OIS-augmentation improves the identification of interest rate expectations.

6.2.2 Risk-Neutral Yields and Short-Horizon Survey Expectations

As further evidence in support of this claim, I compare the model-implied interest rate expectations to short-horizon survey expectations. The preferred GADTSM(s) should also be able to reasonably capture the qualitative and quantitative evolution of comparable-horizon survey expectations. Against this metric, I find that the OIS-augmented model provides superior overall estimates of short-term interest rate expectations, in comparison to all other models.

I compare the estimated 1.5, 4.5, 7.5, 10.5 and 13.5-month risk-neutral yields to corresponding-horizon survey expectations. I calculate approximate short-term interest rate expectations using SPF data for the median expectation of the 3-month T-Bill rate for the remainder of the current quarter, and in the first, second, third and fourth quarters ahead. A complete description of how these expectations are approximated is presented in appendix B. To compare the estimated risk-neutral yields to these survey expectations, I calculate the RMSE of the risk-neutral yields *vis-à-vis* the corresponding-horizon survey expectation on survey submission deadline dates.⁴⁴

Table 2 shows that, on a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates at each horizon. By this metric, the OLS/ML and bias-corrected models provide the most inferior estimates of future short-term interest rate at all three horizons.

At the 4.5, 7.5, 10.5 and 13.5-month horizons the 4-OIS-augmented model provides the superior fit of survey expectations. Strikingly, at the 10.5 and 13.5-month horizons, the RMSE fit of the OLS/ML and bias-corrected models are around three times the RMSE fit of the 4-OIS-augmented model. Although the 3-OIS-augmented model provides the lowest RMSE fit for the 1.5-month survey expectation, the RMSE fit of the 4-OIS-augmented model is only 0.85 basis points higher at this horizon. In contrast, at the 10.5-month horizon the RMSE fit of the 4-OIS-augmented model is 2.34 basis points lower than the 3-OIS-augmented model.

Surprisingly, the survey-augmented model, which uses the same SPF survey expectations as an input to estimation, does not provide a superior fit for these expectations at any horizon *vis-à-vis* the OIS-augmented models. This supports the claim that quarterly frequency survey expectations are not sufficient for the accurate identification of higher frequency interest rate expectations within a GADTSM framework. Nevertheless, the RMSE fit of the survey-augmented

⁴⁴There are two caveats to this comparison which help to explain small differences between survey expectations and risk-neutral yields. First, the expectational horizons of risk-neutral yields and the T-Bill expectations do not exactly correspond, because the latter 3-month T-Bill rate expectations still reflect expected developments up to 3 months beyond the horizon. Second, 3-month T-Bill rates are on a discount basis, whereas risk-neutral yields are quoted on a yield basis.

Table 2: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 1.5, 4.5, 7.5, 10.5 and 13.5-Month Survey Expectations

Sample: January 2002 to December 2016					
RMSE vs. Survey Expectation at Different Horizons					
Model	1.5-Month	4.5-Month	7.5-Month	10.5-Month	13.5-Month
OLS/ML	0.1853	0.2005	0.2700	0.3639	0.4744
Bias-Corrected	0.1857	0.1979	0.2643	0.3570	0.4686
Survey	0.1749	0.1661	0.2216	0.3087	0.4135
3-OIS	0.1642	0.1437	0.1514	0.1545	0.1634
4-OIS	0.1727	0.1354	0.1199	0.1311	0.1577

Note: RMSE of the risk-neutral yields from each of the five GADTSMs in comparison to approximated *Survey of Professional Forecasters* survey expectations, using estimated risk-neutral yields on SPF deadline dates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The construction of the survey expectation approximations is described in appendix B. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

model is superior to the fit of both the OLS/ML and bias-corrected models at all horizons, supporting the claim that augmentation of GADTSMs with additional information can aid the identification of risk-neutral yields.

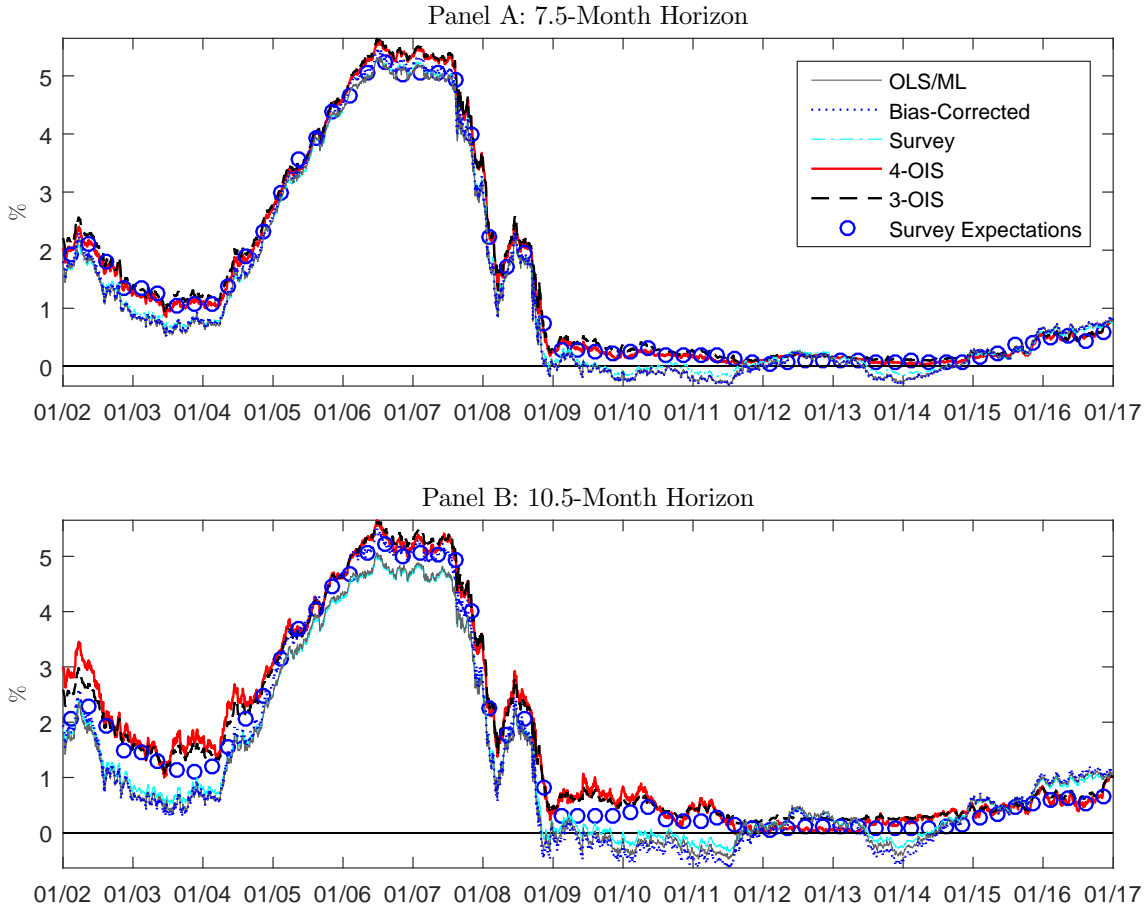
Figure 7 graphically illustrates the evolution of estimated risk-neutral yields and the approximated survey expectations at the 7.5 and 10.5-month horizons — the plots for the 1.5, 4.5 and 13.5-month horizons are qualitatively similar. Three observations follow.

First, between 2002 and late-2004, the OLS/ML, bias-corrected and survey-augmented models generate estimated risk-neutral yields that persistently fall below the corresponding-horizon survey expectation. In contrast, the estimated risk-neutral yields from the OIS-augmented models closely co-move with the approximated survey expectations during this period, especially at the 7.5-month horizon. This corroborates with the comparison of risk-neutral forward yields and federal funds future-implied interest rate expectations in section 6.2.1.

Second, between early-2006 and mid-2007, the risk-neutral yields from the OIS-augmented models exceed interest rate expectations implied by surveys. During this short period, the 7.5-month risk-neutral yields from the OLS/ML, bias-corrected and survey-augmented models more closely align with survey expectations. Although this is not true at the 10.5-month horizon, where the OIS-augmented and bias-corrected models perform best in this period. Recall from figure 6 that the risk-neutral forward yields from the OIS-augmented models closely align with federal funds futures-implied interest rate expectations during this period.

Third, as in section 6.2.1, the GADTSMs offer markedly different estimates of interest rate expectations from late-2008 to late-2011. During this period, the 7.5 and 10.5-month risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying, counter-factually, that investors expected future short-term interest rates to fall negative. From late-2011 to mid-2012, the risk-neutral yields from the OLS/ML and bias-corrected models rise to peak. Again, this is both contrary to the policy narrative and the survey expectations at

Figure 7: Short-Term Interest Rate Expectations



Note: Estimated 7.5-month (panel A) and 10.5-month (panel B) risk-neutral yields from each of five GADTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral yields to approximated survey expectations of future short-term interest rates over the same horizon. The construction of the survey expectation approximations, using data from the *Survey of Professional Forecasters*, is described in appendix B. All figures are in annualised percentage points.

the time. In contrast, the OIS-augmented models — the 4-OIS-augmented model especially — align closely with survey expectations for much of the post-2008 period.

Overall, the results further support the claim that the OIS-augmentation of GADTSMs improves the identification of interest rate expectations. At short-term horizons, OIS-augmented models provide superior estimates of investors' expectations of future short-term interest rates for much of the 2002-2016 sample.

6.2.3 Risk-Neutral Yields and Long-Horizon Survey Expectations

The expectational horizons considered in the previous sub-section are short-term. However, GADTSMs provide estimates of risk-neutral yields for the whole term structure, at horizons further into the future. This is an important motive for using GADTSMs to estimate interest

Table 3: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 10-Year Survey Expectation

Sample: October 2013 to December 2016	
Model	RMSE vs. 10-Year Expectation, Survey of Primary Dealers
OLS/ML	1.5878
Bias-Corrected	1.8747
Survey	1.5309
3-OIS	0.7831
4-OIS	0.7034

Note: RMSE of the risk-neutral yields from each of the five GADTSMs in comparison to the 10-year survey expectation, using estimated risk-neutral yields on survey deadline dates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The median survey expectation is from the Survey of Primary Dealers, New York Federal Reserve. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

rate expectations, instead of market-based financial measures; market-based financial measures seldom provide accurate measures of investors’ interest rate expectations at horizons in excess of 2 years (see [Lloyd, 2017b](#)).

Within a GADTSM, the 10-year risk-neutral yield on date t provides an estimate for the expected average short-term interest rate for the 10-year period following date t . In general, survey data on these longer-term interest rate expectations are not readily available, making it difficult to systematically test the long-horizon interest rate expectations attained from GADTSMs. However, in recent years, the New York Federal Reserve’s *Survey of Primary Dealers* has asked respondents an increasing number of questions regarding their longer-horizon interest rate expectations.⁴⁵ Specifically, since October 2013, respondents have been asked to: “provide your estimate of the longer run target federal funds rate and your expectation for the average federal funds rate over the next 10 years”.⁴⁶ The latter of these requests corresponds to the information contained within the 10-year risk-neutral yields attained from the GADTSMs: the expectation of the average of the short-term interest rate over a 10-year horizon.

To quantitatively assess the longer-horizon interest rate expectations implied by the GADTSMs, I compare the estimated 10-year risk-neutral yield to the median “expectations for the average federal funds rate over the next 10 years” of survey respondents on the survey deadline dates. Again, I calculate the RMSE fit of the risk-neutral yields *vis-à-vis* the survey expectations.

Table 3 presents the results from this analysis. Although the sample of long-horizon survey expectations is relatively short, including 26 surveys from October 2013 to December 2016, the results support the primary conclusion of this paper: that the OIS-augmented models provide unambiguously superior estimates of future short-term interest rate expectations. The RMSE

⁴⁵The questions and results of these surveys are publicly available from: www.newyorkfed.org/markets/primarydealer_survey_questions.html.

⁴⁶In the surveys, the question preceding this was: “provide your estimate of the most likely outcome (i.e., the mode) for the target federal funds rate or range at the end of each half-year period”.

fit of the OLS/ML, bias-corrected and survey-augmented models are over double the RMSE fit of the 4-OIS-augmented model. Moreover, the RMSE fit of the 4-OIS-augmented is smaller than that of the 3-OIS-augmented model, indicating that longer-horizon OIS rates do help to improve the model’s fit of interest rate expectations at longer tenors.

6.2.4 Daily Changes in GADTSM-Implied Interest Rate Expectations

OIS-augmentation offers benefits for the identification and estimation of interest rate expectations from GADTSMs at a *daily frequency*. As OIS rates are available at a daily frequency, they offer potentially sizeable benefits when estimating the daily frequency evolution of interest rate expectations. To illustrate these benefits, I directly analyse the daily changes in GADTSM-implied risk-neutral yields.

The analysis of daily changes in interest rate expectations is an integral part of historical monetary policy analysis. Most recently, a number of authors have used daily changes in interest rate expectations and term premia to assess the relative efficacy of various interest rate channels of unconventional monetary policies (see [Lloyd, 2017c](#), and the references within). For the OIS-augmented GADTSM to be well-suited to historical policy analysis of this sort, it is important that the risk-neutral yields provide an accurate depiction of the daily frequency evolution of interest rate expectations. Specifically, for the GADTSM-implied interest rate expectations to reasonably reflect the expectations of investors over a comparable horizon at a daily frequency, they should, at the very least, qualitatively match numerical measures of investors’ interest rate expectations. To test this, I compare the sign of daily changes in 3, 6 12 and 24-month risk-neutral yields to the sign of daily changes in comparable-maturity OIS rates.⁴⁷ For the GADTSM to reasonably reflect investors’ expectations, the sign of the daily change in the risk-neutral yield should correspond to the sign of the daily change in the comparable horizon OIS rate. I record the proportion of positive and negative daily changes in OIS rates that are matched in sign by the change in the corresponding-horizon risk-neutral yields. To focus on significant changes in OIS rates, I omit days on which OIS rates changed by less, in absolute value, than one standard deviation of the daily changes in the OIS rate over the whole sample (approx 2-5 basis points). The results are presented in table 4.

The results indicate that the 4-OIS-augmented model provides the best qualitative match for the sign of daily changes in 3, 6, 12 and 24-month OIS rates. For example, the 4-OIS-augmented model is the only to match over 96% of positive daily changes in 1-year OIS rates. Moreover, at the 2-year horizon, the sign of daily changes in the risk-neutral yield from the 4-OIS-augmented model matches 97.99% (98.69%) of positive (negative) OIS rate, around 3 (5) percentage points more than the OLS/ML and bias-corrected models match.

Overall, the results in table 4 are consistent with the claim that the 4-OIS-augmented model best reflects the daily frequency evolution of short-term interest rate expectations.

⁴⁷I use the sign of daily changes in OIS rates because their horizon corresponds exactly to that of the nominal government bond yields I use. Although it may seem somewhat tautological to compare an OIS-augmented GADTSM to OIS rates, previous results indicate that this need not be the case. In table 2, the survey-augmented model does not provide the best fit for the survey-expectations which are used as an input to its estimation.

Table 4: Proportion of Daily Changes in OIS Rates Matched in Sign by the Daily Changes in In-Sample GADTSM Risk-Neutral Yields

Sample: January 2002 to December 2016				
Model	Maturity			
	3-Months	6-Months	1-Year	2-Years
Proportion of Positive Daily Changes Matched				
OLS/ML	84.33%	93.20%	95.10%	94.27%
Bias-Corrected	83.58%	92.72%	95.10%	95.13%
Survey	82.09%	91.26%	92.81%	95.13%
3-OIS	85.82%	92.72%	94.44%	96.28%
4-OIS	85.82%	94.17%	96.08%	97.99%
Proportion of Negative Daily Changes Matched				
OLS/ML	87.21%	93.20%	94.93%	93.18%
Bias-Corrected	86.63%	93.13%	95.22%	93.18%
Survey	86.05%	91.85%	94.93%	96.85%
3-OIS	90.12%	93.99%	94.93%	98.69%
4-OIS	90.70%	93.99%	96.42%	98.69%

Note: Proportion of daily changes in 3, 6, 12 and 24-month OIS rates (in excess of one standard deviation of their daily change in absolute value) that are matched in sign by the daily change in the corresponding maturity GADTSM risk-neutral yield. All proportions are expressed as a percentage to two decimal places. Five GADTSMs are compared: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The highest percentage model at each maturity has been emboldened for ease of reading.

6.3 Explaining the Benefits of OIS-Augmentation

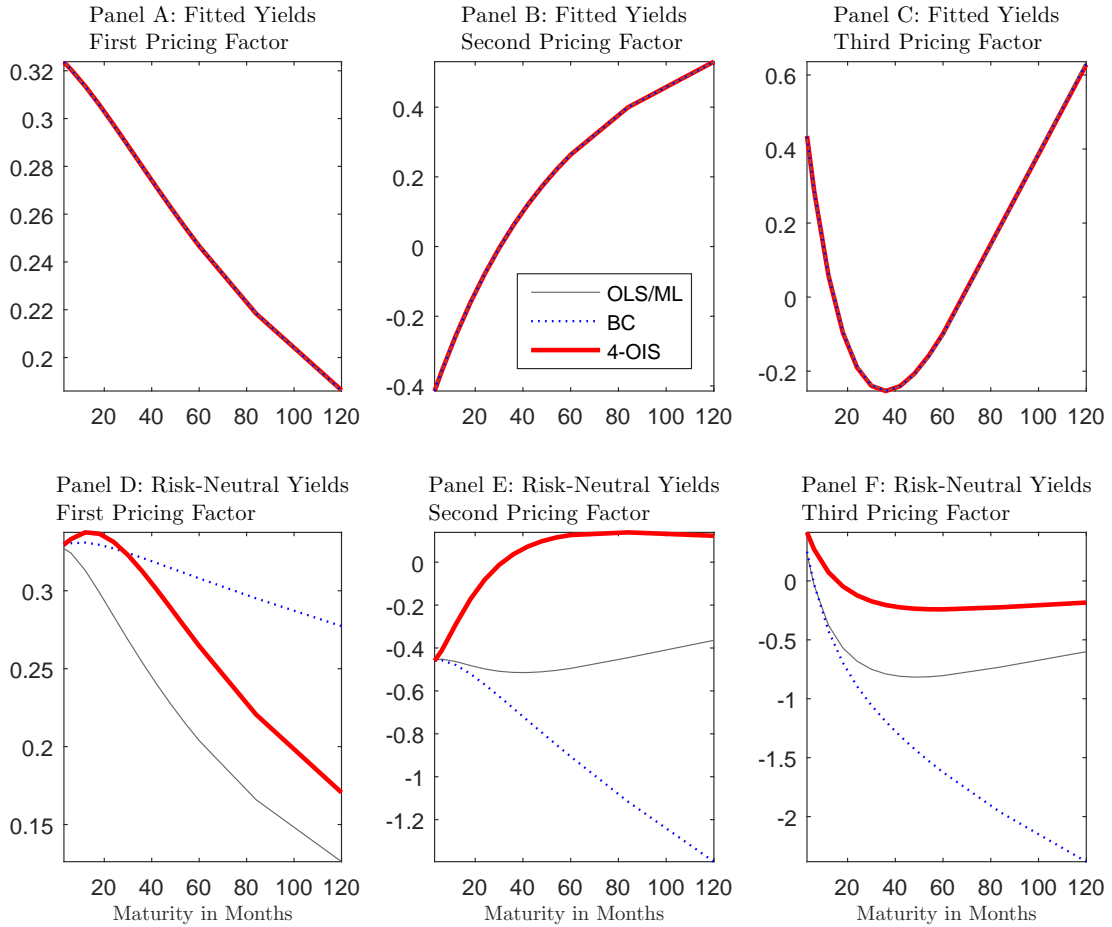
The preceding discussion highlights that OIS-augmented models provide estimates of expected future short-term interest rates that are superior to the OLS/ML, bias-corrected and survey-augmented models. Moreover, within the class of OIS-augmented models considered, the 4-OIS-augmented model, on balance, outperforms the 2 and 3-OIS-augmented models.

Figure 7 highlights that there are differences between the risk-neutral yields from OIS-augmented models and the OLS/ML and bias-corrected models for the whole 2002-2016 sample period. In particular, since late-2008, the risk-neutral yields from the models offer distinctly different qualitative and quantitative predictions for estimated interest rate expectations.

To understand the economic reasons behind these differences, I draw on the canonical description of the first three principal components of bond yields as the level, slope and curvature of the yield curve respectively, together with the model-implied loadings on these factors.⁴⁸ Figure 8 plots these loadings for both the calculation of fitted yields $B_n \equiv -\frac{1}{n}\mathcal{B}_n(\delta_1, \Phi^Q; \mathcal{B}_{n-1})$ (top row) and the risk-neutral yields $\tilde{B}_n \equiv -\frac{1}{n}\mathcal{B}_n(\delta_1, \Phi; \mathcal{B}_{n-1})$ (bottom row) for the 3-month to 10-year maturities. To refine discussion, loadings are presented for the two most inferior models — OLS/ML and bias-corrected — and the most superior — 4-OIS-augmented models.

⁴⁸Because the estimated pricing factors from the three-factor OIS-augmented models almost exactly correspond with the estimated principal components (see figure 4), this economic intuition is valid for these models.

Figure 8: Fitted and Risk-Neutral Yield Factor Loadings



Note: I plot the estimated yield loadings B_n for the fitted and risk-neutral yields, for each the three pricing factors (level, slope and curvature respectively), from the OLS/ML, bias corrected and 4-OIS-augmented models estimated with three factors from January 2002 to December 2016. These coefficients can be interpreted as the *ceteris paribus* response of the fitted and risk-neutral bond yields at a given maturity to a contemporaneous shock to the respective pricing factor. The horizontal axis labels denote the maturity, in months. The three models are denoted by: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); and (iii) the 4-OIS-augmented model (4-OIS).

These loadings illustrate the extent to which the fitted and risk-neutral yields react to a one unit shock to a pricing factor at a given maturity, keeping all other pricing factors constant.

Unsurprisingly, the loadings for the fitted yields from the OLS/ML, bias-corrected and 4-OIS-augmented models are almost identical at all maturities, reinforcing the similarities in their fitted yields. The benefits of OIS-augmentation arise from the separate identification of interest rate expectations and term premia, rather than the fitting of actual yields.

However, the loadings for the risk-neutral yields differ at all horizons, helping to explain why the 4-OIS-augmented model is superior as a measure of interest rate expectations, and providing economic reasons for the differences in risk-neutral yields from late-2008 onwards.

From late-2011 to 2013, the risk-neutral yields from the OLS/ML and bias-corrected models rise to a peak during 2012, falling back below zero for a short period from mid-2013 to mid-

2014. The risk-neutral yields from the OIS-augmented models do not peak during 2012. This period was characterised by two notable phenomena. First, the target federal funds rate was at its effective lower bound. Having been set at this level in December 2008, the FOMC were signalling, through forward guidance, that it would be kept at this rate into the future. Second, the Eurozone sovereign debt crisis elevated Eurozone government bond yields. This was associated with a reduction in yields on, comparatively safe, longer-term US government bonds.⁴⁹ During the 2011-2013 period therefore, the US yield curve was characterised by a reduction in its *slope*, with no change in the *level* of short-term interest rates.

Panel E of figure 8 illustrates that a decrease in the slope of the yield curve places upward pressure on estimated risk-neutral yields in the OLS/ML and bias-corrected models at all maturities. That is, decreases in the yield curve slope, for a given level and curvature, tend to be associated with diminished term premia. However, the risk-neutral yields from the 4-OIS-augmented model react less strongly to a change in the yield curve slope, and, at longer-term horizons, a decrease in the yield curve slope will place downward pressure on estimated risk-neutral yields. This helps to explain why the risk-neutral yields from the 4-OIS-augmented do not rise to a peak in mid-2012, while those from the OLS/ML and bias-correct models do. The 4-OIS-augmented model does not exhibit the same peak, because the inclusion of OIS rates in the estimation alters the loading on that pricing factor. This constellation of factor loadings helps to attain risk-neutral yields from the 4-OIS-augmented model that align more closely with survey and market-implied expectations of future short-term interest rates.

6.4 Model-Implied Term Premia

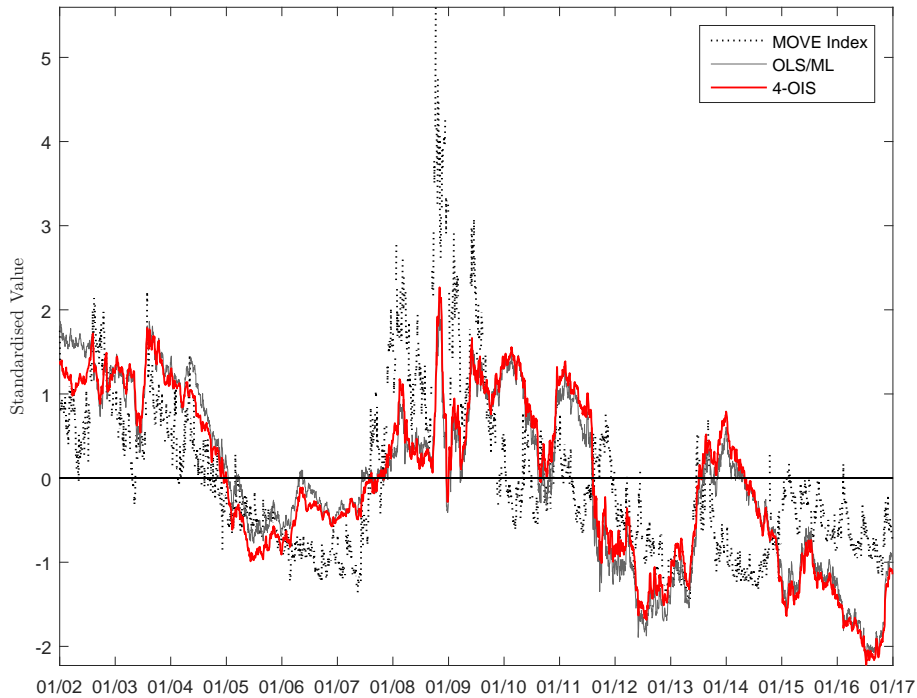
Alongside estimates of expectations of future short-term interest rates, GADTSMs provide estimates for the daily evolution of term premia. Although there is no direct metric against which to compare estimated term premia, [Adrian et al. \(2013\)](#) compare a standardised version of their estimated daily 10-year term premium to a standardised version of the 1-month Merrill Lynch Option Volatility Estimate (MOVE) index. This latter index is a measure of implied volatility from option contracts written on US Treasuring bonds.⁵⁰ Thus, variation in MOVE reflects changes in the risk of holding US Treasuries.

Like [Adrian et al. \(2013\)](#), in figure 9 I plot the standardised z -score estimates of the 10-year term premium from the OLS/ML and 4-OIS-augmented models against the standardised one-month MOVE index. The time series exhibit a strong positive correlation. The correlation coefficient between the standardised 10-year term premium estimate from the 4-OIS-augmented model and the standardised MOVE index is 0.62, marginally higher than the corresponding statistic of 0.60 for the OLS/ML model. This indicates that the estimated term premia from the 4-OIS-augmented model do reflect the risk of holding Treasury bonds.

⁴⁹Longer-term Eurozone interest rates peaked in 2011-2012 as a result of the sovereign debt crisis ([Corsetti, Kuester, Meier, and Müller, 2013, 2014](#)).

⁵⁰Formally, the series used here (and in [Adrian et al., 2013](#)) is defined as a yield curve weighted index of the normalised implied volatility on 1-month Treasury options. It is the weighted average of volatilities on 2, 5, 10 and 30-year bond yields.

Figure 9: Standardised 10-Year Term Premia and Merrill Lynch Option Volatility Estimate (MOVE) Index



Note: I plot the standardised one-month Merrill Lynch Option Volatility Estimate (MOVE) index against the standardised estimates of term premia from (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML), and (ii) the 4-OIS-augmented model (4-OIS). The models are estimated with three factors from January 2002 to December 2016.

7 Conclusion

Financial market participants and policymakers closely monitor the evolution of interest rate expectations using a wide range of financial market instruments. In this paper, I investigate the informational content in OIS rates for this purpose and document how OIS rates can be used to improve estimates of nominal interest rate expectations, and term premia, attained from the term structure of nominal government bond yields.

Drawing on [Lloyd \(2017b\)](#), I report that OIS rates provide an accurate measure of investors' future interest rate expectations out to a 2-year horizon. I then present an OIS-augmented GADTSM for estimating the daily frequency evolution of interest rate expectations, out to a 10-year horizon, that explicitly accounts for the payoff structure in OIS contracts. Most existing arbitrage-free GADTSMs use information in the term structure of nominal government bond yields to identify both expectations of the future path of short-term interest rates and term premia. Numerous authors have drawn attention to an informational insufficiency in the estimation of these models. [Kim and Orphanides \(2012\)](#) propose survey-augmented GADTSMs as a solution to this problem. However, survey forecasts of future interest rates are only available at a low frequency (quarterly or monthly, at best) and reflect investors' expectations of future short-term interest rates for a window of time in the future (e.g. one, two, three or four quarters

ahead). OIS rates, on the other hand, are available at a daily frequency and have a horizon that aligns exactly with those of the zero-coupon nominal government bonds used in the estimation of GADTSMs. The term structure of OIS rates can therefore be readily added to a GADTSM for nominal government bond yields. I show that augmenting the GADTSM with OIS rates provides additional information, specifically related to future interest rate expectations, that can help better identify the evolution of these expectations. Using 3 to 24-month OIS rates in an arbitrage-free GADTSM enables the estimation of future short-term interest rate expectations for the whole term structure — from 3 months to 10 years. Estimates of interest rate expectations from OIS-augmented GADTSMs are superior to those from existing GADTSMs. In particular, short and long-horizon in-sample OIS-augmented risk-neutral yields match patterns in federal funds futures rates and survey expectations. These time series also match qualitative daily patterns exhibited by financial market instruments. This implies that OIS-augmented GADTSMs are well suited for daily frequency policy analysis. Thus, OIS-augmented GADTSMs provide reliable and policy-relevant estimates of interest rate expectations along the whole term structure.

Additionally, this paper highlights the need to test the performance of GADTSMs in a range of dimensions — for example accuracy of fitted yields, risk-neutral yields and term premia — before applying them to analysis of monetary policy. This paper proposes a battery of such tests for future research.

The contribution of this paper extends beyond the GADTSM-literature. For example, the OIS-augmented GADTSM can be applied to better understand the transmission of monetary policy. [Lloyd \(2017c\)](#) uses estimates of interest rate expectations from the OIS-augmented model to assess the effect of US unconventional monetary policy — large-scale asset purchases and forward guidance — on longer-term interest rates. [Lloyd \(2017c\)](#) compares the implications from the OIS-augmented model to the bias-corrected and survey-augmented models, and demonstrates that the use of more accurate estimates of interest rate expectations (from the OIS-augmented model) can have dramatic implications for resulting conclusions, overturning existing results. [Lloyd \(2017c\)](#) concludes that US longer-term interest rates fell on US large-scale asset purchase and forward guidance announcement days, with falls in interest rate expectations, not term premia, explaining the majority of this.

To conclude, OIS rates accurately reflect investors' near-term expectations of future short-term interest rates, providing useful information for improved identification of interest rate expectations and term premia at a range of horizons in arbitrage-free GADTSMs.

Appendix

A Data Sources

The data in section 2 was from the following sources:

Table 5: Data Sources - Average Excess Return Regressions - Section 2

Data Series	Description and Source
- US OIS Rates	<i>Bloomberg</i> , with codes: USSOA 1 month; USSOB 2 months; ... ; USSOK 11 months; USSO1 1 year; USSO1C 15 months; USSO1F 18 months; USSO1I 21 months; USSO2 2 years; and USSO3 3 years.
- US Effective Federal Funds Rate	Federal Reserve Statistical Release H.15: www.federalreserve.gov/releases/h15/data.htm .
- US Survey Forecasts of the 3-Month T-Bill Rate	<i>Survey of Professional Forecasters</i> , available from the Federal Reserve Bank of Philadelphia website: www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files/TBILL/ . The survey deadline dates are available here: www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/spf-release-dates.txt .

The data for the GADTSM was from the following sources:

Table 6: Data Sources - GADTSM - Section 6

Data Series	Description and Source
- US Treasury Bill Rates	Federal Reserve Statistical Release H.15: www.federalreserve.gov/releases/h15/data.htm .
- US Zero-Coupon Treasury Yields	Gürkaynak, Sack, and Wright (2007a) , the updated data from which is available here: www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html .
- US Federal Funds Futures Rates	<i>Bloomberg</i> with codes: FF2, which settles based on the 1st full calendar month in the future; FF3, which settles based on the 2nd full calendar month in the future; ...; FF12, which settles based on the 11th calendar month in the future. And www.quandl.com/data/OFDP/FUTURE_FF_X , where X should be replaced by the horizon of the contract in months.
- US OIS Rates	<i>Bloomberg</i> . See table 5 for detailed source information.
- US Survey Forecasts of the 3-Month T-Bill Rate	See table 5 above.
- Merrill Lynch Option Volatility Estimate (MOVE)	<i>Bloomberg</i> , with the code MOVE Index . This is a yield curve weighted index of the normalised implied volatility on 1-month Treasury options. It is a weighted average of volatilities on the current US 2, 5, 10 and 30 year government notes.

B Approximated Survey Forecasts

In this section, I present the formal details underlying the survey forecast approximation presented in figure 1 and section 6.2.2 using data from the *Survey of Professional Forecasters* (SPF) at the Federal Reserve Bank of Philadelphia. The survey is published every quarter and reports forecasters' median expectations of the average 3-month T-Bill rate over a specified time period: the current quarter $i_{t|t}^{-3m,sur}$; and the first $i_{t+1|t}^{-3m,sur}$, second $i_{t+2|t}^{-3m,sur}$, third $i_{t+3|t}^{-3m,sur}$ and fourth $i_{t+4|t}^{-3m,sur}$ quarters subsequent to the current one, where t denotes the current quarter. All quantities are plotted on the survey submission deadline dates.

To construct a geometric approximation for the average expectation of the 3-month T-Bill rate over the 3-months following the deadline date, I construct an equally weighted geometric average of the median expectation of the 3-month rate for the current and the subsequent quarter. An equal weighting is made possible because the survey deadline date lies approximately halfway through the 'current' quarter. I use a geometric average to facilitate direct comparison with OIS contracts, which have a geometric structure.

To achieve this, I first use the survey expectation for the average 3-month T-Bill rate over the current quarter $i_{t|t}^{-3m,sur}$ and the realised average of the 3-month T-Bill rate in the current quarter up to the SPF deadline date $i_t^{-3m,real}$ to approximate the survey expectations for the average 3-month T-Bill rate over the remainder of the current quarter, denoted $i_{t+|t}^{-3m,sur}$. This is calculated from the following expression:

$$i_{t|t}^{-3m,sur} = \frac{1}{2}i_t^{-3m,real} + \frac{1}{2}i_{t+|t}^{-3m,sur}$$

For figure 1, I calculate the average survey expectation of the 3-month T-Bill rate over the 3, 6 and 12 months following the SPF deadline date. To calculate the average survey expectation of the 3-month T-Bill rate over the three months from the SPF deadline date t , $i_{t|t}^{3m,sur}$, I use the approximation:

$$i_{t|t}^{3m,sur} = \left[\left(1 + \frac{i_{t+|t}^{-3m,sur}}{100} \right)^{\frac{1}{2}} \times \left(1 + \frac{i_{t+1|t}^{-3m,sur}}{100} \right)^{\frac{1}{2}} - 1 \right] \times 100$$

where $i_{t|t}^{3m,sur}$, $i_{t+|t}^{-3m,sur}$ and $i_{t+1|t}^{-3m,sur}$ are all reported in percentage points.

The average expectation of the 3-month T-Bill rate over the six months following the deadline date t , $i_{t|t}^{6m,sur}$, is approximated using a similar geometric *weighted* average procedure: the expectation of the 3-month rate for the remainder of the current quarter and second quarter ahead are both given weights of 1/4; and the first-quarter-ahead expectation has weight 1/2. Mathematically, this is written as:

$$i_{t|t}^{6m,sur} = \left[\left(1 + \frac{i_{t+|t}^{-3m,sur}}{100} \right)^{\frac{1}{4}} \times \left(1 + \frac{i_{t+1|t}^{-3m,sur}}{100} \right)^{\frac{1}{2}} \times \left(1 + \frac{i_{t+2|t}^{-3m,sur}}{100} \right)^{\frac{1}{4}} - 1 \right] \times 100$$

The average expectation of the 3-month T-Bill rate over the year following the submission date t , $i_{t|t}^{1y,sur}$, is approximated by a geometric weighted average of the remainder of the current quarter and first, second, third and fourth quarter ahead expectations, of the form:

$$i_{t|t}^{1y,sur} = \left[\left(1 + \frac{\bar{r}_{t+|t}^{3m,sur}}{100} \right)^{\frac{1}{8}} \times \left(1 + \frac{\bar{r}_{t+1|t}^{3m,sur}}{100} \right)^{\frac{1}{4}} \times \left(1 + \frac{\bar{r}_{t+2|t}^{3m,sur}}{100} \right)^{\frac{1}{4}} \right. \\ \left. \times \left(1 + \frac{\bar{r}_{t+3|t}^{3m,sur}}{100} \right)^{\frac{1}{4}} \times \left(1 + \frac{\bar{r}_{t+4|t}^{3m,sur}}{100} \right)^{\frac{1}{8}} - 1 \right] \times 100$$

In section 6.2.2, I calculate the average expectation of the 3-month T-Bill rate over the 1.5, 4.5, 7.5, 10.5 and 13.5 months following the deadline date. To do this, I construct weighted geometric averages of the median expectation of the 3-month rate for the current and subsequent quarters. The weighting, which facilitates direct comparison of the survey and risk-neutral yield-implied expectations, is made possible because the survey deadline date lies approximately halfway through the ‘current’ quarter. Mathematically, the average survey expectation of the 3-month T-Bill rate over the 1.5, 4.5, 7.5, 10.5 and 13.5 months from the deadline date t are given by:

$$i_{t|t}^{1.5m,sur} = \bar{r}_{t+|t}^{3m,sur}$$

$$i_{t|t}^{4.5m,sur} = \left[\left(1 + \frac{\bar{r}_{t+|t}^{3m,sur}}{100} \right)^{\frac{1}{3}} \times \left(1 + \frac{\bar{r}_{t+1|t}^{3m,sur}}{100} \right)^{\frac{2}{3}} - 1 \right] \times 100$$

$$i_{t|t}^{7.5m,sur} = \left[\left(1 + \frac{\bar{r}_{t+|t}^{3m,sur}}{100} \right)^{\frac{1}{5}} \times \left(1 + \frac{\bar{r}_{t+1|t}^{3m,sur}}{100} \right)^{\frac{2}{5}} \times \left(1 + \frac{\bar{r}_{t+2|t}^{3m,sur}}{100} \right)^{\frac{2}{5}} - 1 \right] \times 100$$

$$i_{t|t}^{10.5m,sur} = \left[\left(1 + \frac{\bar{r}_{t+|t}^{3m,sur}}{100} \right)^{\frac{1}{7}} \times \left(1 + \frac{\bar{r}_{t+1|t}^{3m,sur}}{100} \right)^{\frac{2}{7}} \times \left(1 + \frac{\bar{r}_{t+2|t}^{3m,sur}}{100} \right)^{\frac{2}{7}} \right. \\ \left. \times \left(1 + \frac{\bar{r}_{t+3|t}^{3m,sur}}{100} \right)^{\frac{2}{7}} - 1 \right] \times 100$$

$$i_{t|t}^{13.5m,sur} = \left[\left(1 + \frac{\bar{r}_{t+|t}^{3m,sur}}{100} \right)^{\frac{1}{9}} \times \left(1 + \frac{\bar{r}_{t+1|t}^{3m,sur}}{100} \right)^{\frac{2}{9}} \times \left(1 + \frac{\bar{r}_{t+2|t}^{3m,sur}}{100} \right)^{\frac{2}{9}} \right. \\ \left. \times \left(1 + \frac{\bar{r}_{t+3|t}^{3m,sur}}{100} \right)^{\frac{2}{9}} \times \left(1 + \frac{\bar{r}_{t+4|t}^{3m,sur}}{100} \right)^{\frac{2}{9}} - 1 \right] \times 100$$

C Baseline Gaussian Affine Dynamic Term Structure Model

C.1 Bond Pricing Using the Risk-Adjusted Probability Measure \mathbb{Q}

To guarantee the existence of a risk-adjusted probability measure \mathbb{Q} , under which the bonds are priced, no-arbitrage is imposed (Harrison and Kreps, 1979). The risk-adjusted probability measure \mathbb{Q} is defined such that the price V_t of any asset that does not pay any dividends at time $t + 1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)V_{t+1}]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the \mathbb{Q} probability measure. Thus, with no-arbitrage, the price of an n -day zero-coupon bond must satisfy the following relation:

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \quad (12)$$

Using this, it is possible to show that the nominal bond price is an exponentially affine function of the pricing factors:

$$P_{t,n} = \exp(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t) \quad (13)$$

such that the corresponding continuously compounded yield $y_{t,n}$ is affine in the pricing factors:

$$y_{t,n} = -\frac{1}{n} \ln(P_{t,n}) = A_n + B_n \mathbf{x}_t \quad (20)$$

where $A_n \equiv -\frac{1}{n} \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $B_n \equiv -\frac{1}{n} \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1})$.

To attain recursive expressions for \mathcal{A}_n and \mathcal{B}_n :

$$\begin{aligned} \mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t &= \ln P_{t,n} \\ &= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \\ &= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \mathbf{x}_{t+1})] \\ &= \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\delta_0 - \boldsymbol{\delta}'_1 \mathbf{x}_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right]\right)\right] \\ &= -(\delta_0 + \boldsymbol{\delta}'_1 \mathbf{x}_t) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{x}_t\right] + \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right] \\ &= -(\delta_0 + \boldsymbol{\delta}'_1 \mathbf{x}_t) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{x}_t\right] + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}'_{n-1} \\ &= \left\{-\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}'_{n-1} + \mathcal{B}_{n-1} \boldsymbol{\mu}^{\mathbb{Q}}\right\} + \left\{-\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1} \boldsymbol{\Phi}^{\mathbb{Q}}\right\} \mathbf{x}_t \end{aligned}$$

using (13) in the third line, (6) and (9) in the fourth line, and using the property of the log-normal distribution in conjunction with the fact that $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} | \mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$ to write the expression $\ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right]$ as $\frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}'_{n-1}$ in the sixth line.

By the method of undetermined coefficients, the recursive definitions for the scalar $\mathcal{A}_n \equiv \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and the $1 \times K$ vector $\mathcal{B}_n \equiv \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1})$ follow from

the final line:

$$\mathcal{A}_n = -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\Sigma\Sigma'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} \quad (21)$$

$$\mathcal{B}_n = -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} \quad (22)$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}'_K$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

C.2 Bond Pricing Using the Pricing Kernel and the Actual Probability Measure \mathbb{P}

Under the actual \mathbb{P} probability measure, the bond price is given by equation (11):

$$P_{t,n} = \mathbb{E}_t [M_{t+1}P_{t+1,n-1}]$$

where this expectation is taken under the \mathbb{P} measure.

Using this, it is also possible to show that the nominal bond price is an exponentially affine function of the pricing factors, as in (13). To attain recursive expressions for \mathcal{A}_n and \mathcal{B}_n :

$$\begin{aligned} \mathcal{A}_n + \mathcal{B}_n\mathbf{x}_t &= \ln P_{t,n} \\ &= \ln \mathbb{E}_t [M_{t+1}P_{t+1,n-1}] \\ &= \ln \mathbb{E}_t \left[\exp \left(-it - \frac{1}{2}\boldsymbol{\lambda}'_t\boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t\boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1} \right) \right] \\ &= \ln \mathbb{E}_t \left[\exp \left(-\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t - \frac{1}{2}(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)'(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t) \right. \right. \\ &\quad \left. \left. - (\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)'\boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \Sigma\boldsymbol{\varepsilon}_{t+1}) \right) \right] \\ &= -\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t - \frac{1}{2}(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)'(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t) \\ &\quad + \ln \mathbb{E}_t \left[\exp \left((-\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)' + \mathcal{B}_{n-1}\Sigma \right) \boldsymbol{\varepsilon}_{t+1} \right] \\ &= -\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t - \frac{1}{2}(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)'(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t) \\ &\quad + \frac{1}{2} \left((-\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t)' + \mathcal{B}_{n-1}\Sigma \right) \left(-\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t \right)' + \mathcal{B}_{n-1}\Sigma' \\ &= -\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t + \mathcal{A}_{n-1} - \mathcal{B}_{n-1}\Sigma(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t) \\ &\quad + \frac{1}{2}\mathcal{B}_{n-1}\Sigma\Sigma'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t) \end{aligned}$$

using (7) and (13) in the third line, and (5), (6) and (8) in the fourth line.

By the method of undetermined coefficients, the recursive definitions for the scalar \mathcal{A}_n and the $1 \times K$ vector \mathcal{B}_n follow from the final line:

$$\mathcal{A}_n = -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\Sigma\Sigma'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}(\boldsymbol{\mu} - \Sigma\boldsymbol{\lambda}_0) \quad (23)$$

$$\mathcal{B}_n = -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1}(\boldsymbol{\Phi} - \Sigma\boldsymbol{\Lambda}_1) \quad (24)$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}'_K$, where $\mathbf{0}'_K$ is a $K \times 1$ vector of zeros.

Comparing (21) and (22) with (23) and (24) yields the relationship between \mathbb{P} and \mathbb{Q} parameters:

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \quad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1.$$

C.3 Risk-Neutral Yields

The risk-neutral yield on an n -day bond reflects the yield that would prevail if investors were risk-neutral. That is, the risk-neutral yield corresponds to that which would arise under the actual probability measure \mathbb{P} .

The risk-neutral bond price $\tilde{P}_{t,n}$ is of the form:

$$\tilde{P}_{t,n} = \mathbb{E}_t \left[\exp(-i_t) \tilde{P}_{t+1,n-1} \right] \quad (25)$$

and can be shown to be an exponentially affine function of the pricing factors:

$$\tilde{P}_{t,n} = \exp(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t) \quad (26)$$

where $\mathcal{A}_n \equiv \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $\mathcal{B}_n \equiv \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1})$. Thus, the risk-neutral yield, $\tilde{y}_{t,n} = -\frac{1}{n} \ln \tilde{P}_{t,n}$, is affine in the pricing factors:

$$\tilde{y}_{t,n} = \tilde{A}_n + \tilde{B}_n \mathbf{x}_t \quad (27)$$

where $\tilde{A}_n = -\frac{1}{n} \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $\tilde{B}_n = -\frac{1}{n} \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1})$.

To attain the recursive expressions for \tilde{A}_n and \tilde{B}_n , note that from equation (26):

$$\begin{aligned} \tilde{y}_{t,n} &= -\frac{1}{n} \ln \mathbb{E}_t \left[\exp \{ -i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \mathbf{x}_{t+1} \} \right] \\ \tilde{A}_n + \tilde{B}_n \mathbf{x}_t &= -\frac{1}{n} \ln \mathbb{E}_t \left[\exp \{ -(\delta_0 + \boldsymbol{\delta}'_1 \mathbf{x}_t) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} [\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}] \} \right] \\ &= -\frac{1}{n} \left[\left\{ -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}'_{n-1} + \mathcal{B}_{n-1} \boldsymbol{\mu} \right\} + \{ -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1} \boldsymbol{\Phi} \} \mathbf{x}_t \right] \end{aligned}$$

using (25) and (26) in the first line, and (6) and (5) in the second line. The expectation is taken under the actual probability measure \mathbb{P} . By the method of undetermined coefficients, it follows that:

$$\tilde{y}_{t,n} = \tilde{A}_n + \tilde{B}_n \mathbf{x}_t$$

where $\tilde{A}_n = -\frac{1}{n} \mathcal{A}_n(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1})$ and $\tilde{B}_n = -\frac{1}{n} \mathcal{B}_n(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1})$.

D Overnight Indexed Swap Augmentation

To calculate the loadings for the OIS observation equation, first note that the *annualised* floating leg of a j -day OIS contract, with trade day t and starting day $t + 1$, is given by:

$$i_{t,t+j}^{flt} = \left(\left[\prod_{k=1}^j (1 + \gamma_{t+k} i_{t+k}) \right] - 1 \right) \times \frac{T_{yr}}{j}$$

where γ_{t+k} is an accrual factor, which is set to $1/T_{yr}$ for all time periods, and $T_{yr} = 252$ is the number of trading days in a year.⁵¹ i_t is the one-period short-term floating interest rate (6) used as the reference rate for the swap — the effective federal funds rate. Rearranging this, taking logs, and using a first-order Taylor approximation around $x = 0$ such that $\ln(1 + x) \approx x$, yields:

$$i_{t,t+k}^{flt} \approx \left[\sum_{k=1}^j \gamma_{t+k} i_{t+k} \right] \times \frac{T_{yr}}{j} \quad (28)$$

Therefore, under the expectations hypothesis, the OIS rate $i_{t,t+j}^{ois}$ will be:

$$i_{t,t+j}^{ois} = \mathbb{E}_t \left[\sum_{k=1}^j \gamma_{t+k} i_{t+k} \right] \times \frac{T_{yr}}{j} \quad (29)$$

For a one-period OIS contract, $j = 1$:

$$\begin{aligned} i_{t,t+1}^{ois} &= \mathbb{E}_t [\gamma (\delta_0 + \delta'_1 \mathbf{x}_{t+1})] \times T_{yr} \\ &= \mathbb{E}_t [\gamma (\delta_0 + \delta'_1 [\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}])] \times T_{yr} \\ &= (\delta_0 + \delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_t) \end{aligned}$$

where $\gamma \equiv 1/T_{yr}$, and noting that $\mathbb{E}_t [\boldsymbol{\varepsilon}_{t+1}] = \mathbf{0}_K$ in the second line.

For a two period OIS contract, $j = 2$, the expectations hypothesis requires that:

$$\begin{aligned} i_{t,t+2}^{ois} &= \mathbb{E}_t [\gamma (\delta_0 + \delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_t) + \gamma (\delta_0 + \delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_{t+1})] \times (T_{yr}/2) \\ &= \mathbb{E}_t [\gamma (\delta_0 + \delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_t) + \gamma (\delta_0 + \delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} [\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1}])] \times (T_{yr}/2) \\ &= \frac{1}{2} (2\delta_0 + 2\delta'_1 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_t + \delta'_1 \boldsymbol{\Phi}^2 \mathbf{x}_t) \end{aligned}$$

For a three period OIS contract, $j = 3$, the same steps as above yield the following expression:

$$i_{t,t+3}^{ois} = \frac{1}{3} (3\delta_0 + 3\delta'_1 \boldsymbol{\mu} + 2\delta'_1 \boldsymbol{\Phi} \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi}^2 \boldsymbol{\mu} + \delta'_1 \boldsymbol{\Phi} \mathbf{x}_t + \delta'_1 \boldsymbol{\Phi}^2 \mathbf{x}_t + \delta'_1 \boldsymbol{\Phi}^3 \mathbf{x}_t)$$

⁵¹For the term structure model, the accrual and annualisation factors use the convention that there are 252 business trading days in a year, as opposed to the market quoting convention of 360 days used in section 2. Given that daily yield data is only available on 252 days per year, I adopt this convention to ensure that the *horizon* for each OIS rate corresponds to their actual maturity date and that of a corresponding maturity zero-coupon bond. This convention is also adopted for daily frequency term structure estimation by, amongst others, [Bauer and Rudebusch \(2014\)](#).

The continued iteration can be summarised by the following expressions:

$$i_{t,t+j}^{ois} = A_j^{ois} + B_j^{ois} \mathbf{x}_t \quad (18)$$

where $A_j^{ois} \equiv \frac{1}{j} \mathcal{A}_j^{ois} (\delta_0, \delta_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{j-1}^{ois}, \mathcal{B}_{j-1}^{ois})$ and $B_j^{ois} = \frac{1}{j} \mathcal{B}_j^{ois} (\delta_1, \boldsymbol{\Phi}; \mathcal{B}_{j-1}^{ois})$ are recursively defined as:

$$\begin{aligned} \mathcal{A}_j^{ois} &= \delta_0 + \delta_1' \boldsymbol{\mu} + \mathcal{A}_{j-1}^{ois} + \mathcal{B}_{j-1}^{ois} \boldsymbol{\mu} \\ \mathcal{B}_j^{ois} &= \delta_1' \boldsymbol{\Phi} + \mathcal{B}_{j-1}^{ois} \boldsymbol{\Phi} \end{aligned}$$

where $\mathcal{A}_0^{ois} = 0$ and $\mathcal{B}_0^{ois} = \mathbf{0}'_K$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

I attain an affine expression for OIS rates (18), with loadings that are additive and recursive, because I use a first-order Taylor approximation of $i_{t,t+j}^{flt}$ in (28). This ensures that OIS rates can be included in the GADTSM in a similar manner to bond yields, which are also affine in the pricing factors \mathbf{x}_t . In reality, because the floating leg of an OIS contract is compounded, a Jensen's inequality term would be expected in the OIS pricing expression, representing a term premium in OIS rates. The first-order Taylor approximation of $i_{t,t+j}^{ois}$ (28) prevents a Jensen's inequality term from arising, and considerably simplifies the expressions for loadings, \mathcal{A}_j^{ois} and \mathcal{B}_j^{ois} , ensuring that they are additive and recursive. I circumvent this potential problem by only using OIS rates which I demonstrate have statistically insignificant *ex post* excess returns, such that any Jensen's inequality term should be negligible. Furthermore, I continue to admit measurement error in OIS rates in the Kalman filter setup (19) through \mathbf{u}_t^{ois} .

E Estimation Procedure

To identify the unaugmented model described in section 3, I use the normalisation scheme proposed by Joslin et al. (2011). The Joslin et al. (2011) normalisation fosters faster convergence to the global optimum of the model's likelihood function than other identification schemes for two reasons. First, this normalisation allows for the (near) separation of the \mathbb{P} and \mathbb{Q} probability measure likelihood functions, the product of which comprises the overall model likelihood function. Moreover, the Joslin et al. (2011) normalisation reduces the dimensionality of the parameter space. In the baseline, unaugmented model, the parameters governing bond pricing are:

$$\Theta = \left\{ \delta_0, \delta_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma} \right\}$$

The Joslin et al. (2011) normalisation scheme uniquely maps these parameters to a smaller set:

$$\left\{ i_{\infty}^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\Sigma} \right\}$$

where: (i) $i_{\infty}^{\mathbb{Q}}$ is the risk-neutral expectation of the long-run short-term nominal interest rate; (ii) $\boldsymbol{\lambda}^{\mathbb{Q}}$ is a $K \times 1$ vector of the eigenvalues of $\boldsymbol{\Phi}^{\mathbb{Q}}$; and (iii) $\boldsymbol{\Sigma}$ is a lower triangular matrix with positive diagonal entries.

For all the term structure models estimated in this paper I use the convention that there are 252 business days in a year, corresponding to the number of days for which bond yield data exists per year.⁵² To ensure that the *horizon* for each bond corresponds to their actual maturity date, I set the daily horizons of the 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 month yields to $n = 63, 126, 252, 378, 504, 630, 756, 882, 1008, 1134, 1260, 1764$ and 2520 days ($N = 13$). Similarly, the 3, 6, 12 and 24 month OIS rates have the following horizon in days: $j = 63, 126, 252, 504$ ($J = 4$).

E.1 OLS/ML Estimation of the Baseline, Unaugmented GADTSM

Assuming that K portfolios of bonds are priced without error, then the Joslin et al. (2011) normalisation permits the complete separation of the \mathbb{P} and \mathbb{Q} likelihood functions. In this paper, as in many others, I use the first K principal components of the observed bond yields as the set of K portfolios that are priced perfectly (e.g. Joslin et al., 2011). Defining these portfolios $\mathcal{P}_t \equiv W y_t = W y_t^{obs} \equiv \mathcal{P}_t^{obs}$, where W is the principal component weighting matrix and y_t^{obs} is the vector of observed yields, then Joslin et al. (2011) show that the likelihood function for the unaugmented model laid out in section 3.1 is:

$$\mathcal{L} \left(y_t^{obs} | y_{t-1}^{obs}; \Theta \right) = \mathcal{L} \left(y_t^{obs} | \mathcal{P}_t; \boldsymbol{\lambda}^{\mathbb{Q}}, i_{\infty}^{\mathbb{Q}}, \boldsymbol{\Sigma}, \sigma_u \right) \times \mathcal{L} \left(\mathcal{P}_t | \mathcal{P}_{t-1}; \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma} \right)$$

where σ_u is the standard deviation of the measurement error of the N observed yields.

This normalisation admits a two-stage estimation process. First, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ are

⁵²This convention is also adopted for daily frequency term structure estimation by, amongst others, Bauer and Rudebusch (2014).

directly estimable by running OLS on the VAR in equation (5), where $\mathbf{x}_t \equiv \mathcal{P}_t$. Moreover, this provides initial values for the maximum likelihood estimation of the lower triangular elements of the matrix Σ . Second, taking $\{\hat{\boldsymbol{\mu}}, \hat{\Phi}\}$ as given, the parameters $\{i_\infty^Q, \boldsymbol{\lambda}^Q, \Sigma, \sigma_u\}$ can be estimated by maximum likelihood.

E.2 Bias-Corrected Estimation

To estimate the bias-corrected decomposition, I rely entirely on the methodology of [Bauer et al. \(2012, Section 4\)](#). The MATLAB code for this is available here: faculty.chicagobooth.edu/jing.wu/research/zip/brw_table1.zip.

E.3 Survey-Augmentation

To augment the model with survey expectations of future interest rates, I employ Kalman filter-based maximum likelihood estimation. This estimation methodology, using survey expectations, draws most directly on [Guimarães \(2014\)](#).

Like [Guimarães \(2014\)](#), I use survey expectations from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. I use forecasts for the 3 month T-Bill 1, 2, 3 and 4 quarters ahead, available at a quarterly frequency. I augment the model with the survey expectations on the survey submission deadline day.⁵³

The survey-augmented Kalman filter has a similar form to the OIS-augmented setup presented in section 3. The transition equation of the Kalman filter is (5), the vector autoregression for the latent pricing factors under the actual \mathbb{P} probability measure.

On days when the survey forecasts are *not* observed, the observation equation is given by (17). As with the OIS-augmented model, I maintain a homoskedastic form for the yield measurement error.

On days when the S survey forecasts, $s = s_1, s_2, \dots, s_S$, are observed, the observation equation is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{sur} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{sur} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{sur} \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \Sigma_Y & \mathbf{0}_{N \times S} \\ \mathbf{0}_{S \times N} & \Sigma_S \end{bmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^{sur} \end{bmatrix} \quad (30)$$

where, in addition to the definitions of \mathbf{y}_t , \mathbf{A} , \mathbf{B} , Σ_Y and \mathbf{u}_t above, $\mathbf{i}_t^{sur} = [i_{t,s_1}^{sur}, \dots, i_{t,s_S}^{sur}]'$; $\mathbf{A}^{sur} = [A_{s_1}^{sur}, \dots, A_{s_S}^{sur}]'$; $\mathbf{B}^{sur} = [B_{s_1}^{sur}, \dots, B_{s_S}^{sur}]'$; $\mathbf{0}_{S \times N}$ and $\mathbf{0}_{N \times S}$ denote $S \times N$ and $N \times S$ matrices of zeros respectively; and $\mathbf{u}_t^{sur} \sim \mathcal{N}(\mathbf{0}_S, \mathbf{I}_S)$ denotes the survey measurement error, where $\mathbf{0}_S$ is an S -vector of zeros and \mathbf{I}_S is an $S \times S$ identity matrix. As with the yield measurement error, I impose a homoskedastic form for the survey measurement error, such that Σ_S is a $S \times S$ diagonal matrix with common diagonal element σ_s , the standard deviation of the survey measurement error. Appendix C of [Guimarães \(2014\)](#) presents the functional forms for A_s^{sur} and B_s^{sur} , which account for the arithmetic nature of survey expectations.

As with the OIS-augmented model, I estimate the survey-augmented model by using the OLS/ML parameter estimates as initial values for the Kalman filter.

⁵³For survey submission dates that are not business days, I augment the model with survey data on the preceding business day.

E.4 OIS-Augmentation of the GADTSM and Kalman Filtering

When the Kalman filter is used, the assumption that K portfolios of yields are observed without error is no longer made. Instead, all yields (and portfolios thereof) can be observed with error. Consequently, the exact separation of the likelihood function described in section E.1 is no longer applicable. However, the parameter estimates attained from OLS/ML estimation of the unaugmented model do provide initial values for the Kalman filter-based optimisation routine.⁵⁴ Doing so, ensures that computational time is reasonably fast.

⁵⁴Guimarães (2014) follows similar steps to estimate a survey-augmented GADTSM using the Joslin et al. (2011) normalisation scheme.

F Term Structure Results

In this section, I present additional results from the GADTSM estimation.

F.1 Additional Results for the Three-Factor Specification

F.1.1 Model-Implied Fitted Yields

Table 7 presents the root mean square error (RMSE) for the fitted yields from each of the term structure models. The RMSE is presented for each maturity, and the average over all maturities, for the sample period January 2002 to December 2016. The fit, compared to the actual yield, is broadly similar across all six models. Specifically, the average RMSE for each of the models at all thirteen maturities is around 5 basis points, and differs by no more than 0.29 basis points across models. Thus, all models fit actual bond yields similarly well.

Table 7: GADTSM Fit: Root Mean Square Error (RMSE) of the Fitted Yields *vis-à-vis* the Actual Yields

Sample: January 2002 to December 2016						
Maturity	OLS/ML	BC	Survey	2-OIS	3-OIS	4-OIS
3-Months	0.0979	0.0983	0.1028	0.1000	0.1091	0.1076
6-Months	0.0516	0.0513	0.0530	0.0524	0.0489	0.0534
1-Year	0.0714	0.0717	0.0775	0.0774	0.0777	0.0739
18-Months	0.0567	0.0564	0.0591	0.0598	0.0598	0.0605
2-Years	0.0403	0.0396	0.0395	0.0404	0.0396	0.0435
30-Months	0.0240	0.0234	0.0228	0.0237	0.0222	0.0265
3-Years	0.0161	0.0159	0.0181	0.0182	0.0177	0.0179
42-Months	0.0223	0.0223	0.0256	0.0249	0.0262	0.0237
4-Years	0.0313	0.0311	0.0339	0.0328	0.0349	0.0330
54-Months	0.0378	0.0374	0.0393	0.0380	0.0405	0.0401
5-Years	0.0410	0.0403	0.0414	0.0400	0.0425	0.0440
7-Years	0.0273	0.0263	0.0267	0.0265	0.0249	0.0333
10-Years	0.0638	0.0629	0.0609	0.0559	0.0608	0.0558
Average	0.0499	0.0497	0.0517	0.0507	0.0526	0.0525

Note: RMSE of the fitted yields from each of the six three-factor GADTSMs, computed by comparing the model-implied fitted yield to the actual yield on each day. All figures are expressed in annualised percentage points. The six GADTSMs are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); (iii) the survey-augmented model (Survey); (iv) the 2-OIS-augmented model (2-OIS); (v) the 3-OIS-augmented model (3-OIS); and (vi) the 4-OIS-augmented model (4-OIS).

F.1.2 Model-Implied Fitted OIS Rates

Table 8 presents the RMSE for the fitted OIS rates from each of the OIS-augmented term structure models. The RMSE is presented for each maturity for the sample period January 2002 to December 2016. The results demonstrate that the 4-OIS-augmented model provides superior estimates of the 6, 12 and 24-month OIS rates, while the 2-OIS-augmented model provides marginally superior estimates of the 3-month OIS rate. At the 6 and 12-month horizons, the 3-OIS-augmented model is only marginally inferior to the 4-OIS-augmented model. However, at the 2-year horizon, the 4-OIS-augmented model provides a substantial improvement in fit *vis-à-vis* the 3 and 2-OIS-augmented models. The 2-OIS-augmented provides the highest RMSE estimates of 1 and 2-year OIS rates.

Table 8: GADTSM Fit: Root Mean Square Error (RMSE) of Fitted OIS Rates *vis-à-vis* the Actual OIS Rates

Sample: January 2002 to December 2016			
Maturity	2-OIS	3-OIS	4-OIS
3-Months	0.1183	0.1206	0.1305
6-Months	0.0924	0.1088	0.0861
1-Year	0.1585	0.0926	0.0831
2-Year	0.5306	0.2634	0.0985

Note: RMSE of the fitted OIS rates from each of the three OIS-augmented GADTSMs, computed by comparing the model-implied fitted OIS rate to the actual OIS rate on each day. All figures are expressed in annualised percentage points. The three GADTSMs are: (i) the 2-OIS-augmented model (2-OIS); (ii) the 3-OIS-augmented model (3-OIS); and (iii) the 4-OIS-augmented model (4-OIS).

F.1.3 Estimated Pricing Factors and Principal Components

Table 9 presents summary statistics for the estimated principal components of the actual bond yields and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models for the sample period January 2002 to December 2016. The results demonstrate that the principal components and estimated pricing factors evolve similarly, implying that OIS rates do not include any additional information, over and above that in bond yields, of value to the fitting of actual yields. In particular, the summary statistics of the estimated principal components and the estimated pricing factors from the 4-OIS-augmented models are similar.

Moreover, table 9 further demonstrates that the inclusion of different maturities of OIS rate in the term structure model does not appreciably alter estimates of actual bond yields. The summary statistics of the estimated pricing factors from the 4, 3 and 2-OIS-augmented models are all similar. Augmentation of GADTSMs with OIS rates only influences estimated parameters under the actual probability measure \mathbb{P} and thus risk-neutral yields.

Table 9: Estimated Principal Components and Estimated Pricing Factors: Summary Statistics

Summary Statistics	1st Factor	2nd Factor	3rd Factor
Estimated Principal Components			
Mean	0.0725	0.0326	0.0075
Variance	0.0026	0.0001	0.0000
Skewness	0.7771	0.3839	-0.2603
Kurtosis	2.2918	2.2017	2.2441
4-OIS: Estimated Pricing Factors			
Mean	0.0725	0.0326	0.0076
Variance	0.0025	0.0001	0.0000
Skewness	0.7712	0.4043	-0.2515
Kurtosis	2.2792	2.2136	2.2732
3-OIS: Estimated Pricing Factors			
Mean	0.0725	0.0326	0.0075
Variance	0.0025	0.0001	0.0000
Skewness	0.7756	0.3909	-0.2542
Kurtosis	2.2877	2.1886	2.2772
2-OIS: Estimated Pricing Factors			
Mean	0.0725	0.0326	0.0075
Variance	0.0025	0.0001	0.0000
Skewness	0.7756	0.3797	-0.2812
Kurtosis	2.2893	2.1930	2.2152

Note: Summary statistics for the first three estimated principal components from actual yield data and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models. All statistics are reported to four decimal places.

F.1.4 Stability of Estimates

In order to apply the term structure model to policy analysis, it is desirable for estimates of fitted yields, risk-neutral yields and term premia on any given date to be stable across different sample periods. That is, one would like a model that provides similar estimates of interest rate expectations on a given date regardless of the sample period used. In principle, OIS-augmentation can be helpful in this regard. By augmenting the model with additional information about interest rate expectations to solve the identification problem within GADTSMs, the OIS-augmented model should provide estimates of interest rate expectations that vary less with respect to the sample period in comparison to unaugmented models.

To assess the usefulness of the OIS-augmented GADTSM for real-time policy analysis, I compare real-time estimates of fitted and risk-neutral yields to estimates from the same model estimated using three different samples. To focus on monetary policy, I compare estimates on 19 different dates, the first 16 of which were major US unconventional monetary policy

announcement days (see [Lloyd, 2017c](#), table 3).⁵⁵ [Lloyd \(2017c\)](#) demonstrates that many of these announcements significantly affected bond yields. The remaining 3 dates are: 30/04/2013, the end of the month prior to the May 2013 ‘taper tantrum’; 31/12/2015, the end of the term structure sample in [Lloyd \(2017c\)](#); and 31/12/2016.

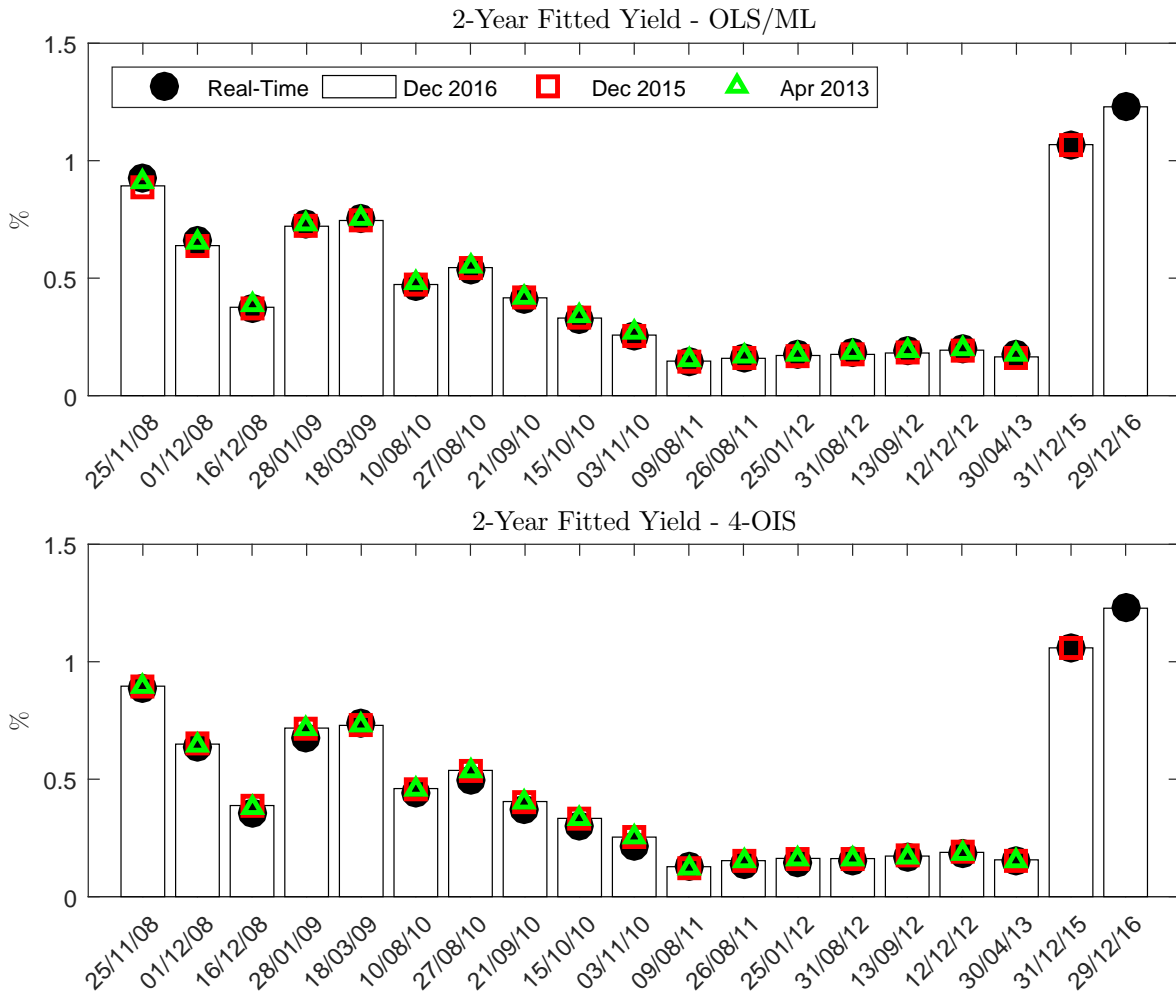
I attain real-time estimates by re-estimating each model using data up to the date of interest. The start date of all real-time samples is January 2002. The three alternative samples all begin in January 2002 and end in April 2013, December 2015 and December 2016, respectively.

Figures 10 and 11 plot estimates of 2-year fitted yields and 2-year risk-neutral yields, respectively, from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on the 19 different dates. The black dots represent the real-time estimates, which are most relevant for monetary policy. The white bars illustrate the 2-year fitted yields and 2-year risk-neutral yields from the January 2002 to December 2016 sample. The red squares plot the same quantities from the January 2002 to December 2015 sample, while the green triangles plot the results from the January 2002 to April 2013 sample. To attain reliable inference from an event study using a GADTSM, estimates of interest rate expectations and term premia on a given date from a desirable model should not vary across sample periods. For example, the estimated influence of the initial announcement of large-scale asset purchases (25/11/2008) on interest rate expectations and term premia should not change significantly as the estimation sample period is extended.

Figure 10 illustrates that real-time estimates of the fitted yield are very similar to those attained using the three longer samples. This is unsurprising, as the identification problem pertains to the risk-neutral yields. However, the top panel of figure 11 demonstrates that the identification problem does generate instability in risk-neutral yield estimates in the unaugmented OLS/ML model. Real-time estimates of the level of interest rate expectations differ substantially from those attained from the three longer samples. Moreover, the estimates from the three longer samples substantially differ from one another. For example, on March 18, 2009, the real-time estimate of the 2-year risk-neutral yield from the OLS/ML model is 111 basis points above the estimate attained from the January 2002 to December 2016 sample, which, in turn, is 29 basis points below the estimate from the January 2002 to April 2013 sample. In contrast, the bottom panel of figure 11 illustrates that estimates of risk-neutral yields from the 4-OIS-augmented model are remarkably stable across samples. Although, there are differences between the real-time estimates and longer-sample estimates for early events that peak at 17 basis points on December 1, 2008 and December 16, 2008, this is likely to be due to parameter instability around 2008-2010. As the sample is extended to include more post-2008 data, the differences between estimates decline. On December 12, 2012, the range of estimates from the 4-OIS-augmented model is just 7 basis points; the corresponding figure for the OLS/ML model is 65 basis points. Moreover, the differences between real-time and longer-sample estimates for early events cannot be explained by small-sample issues, because shorter-sample estimates are

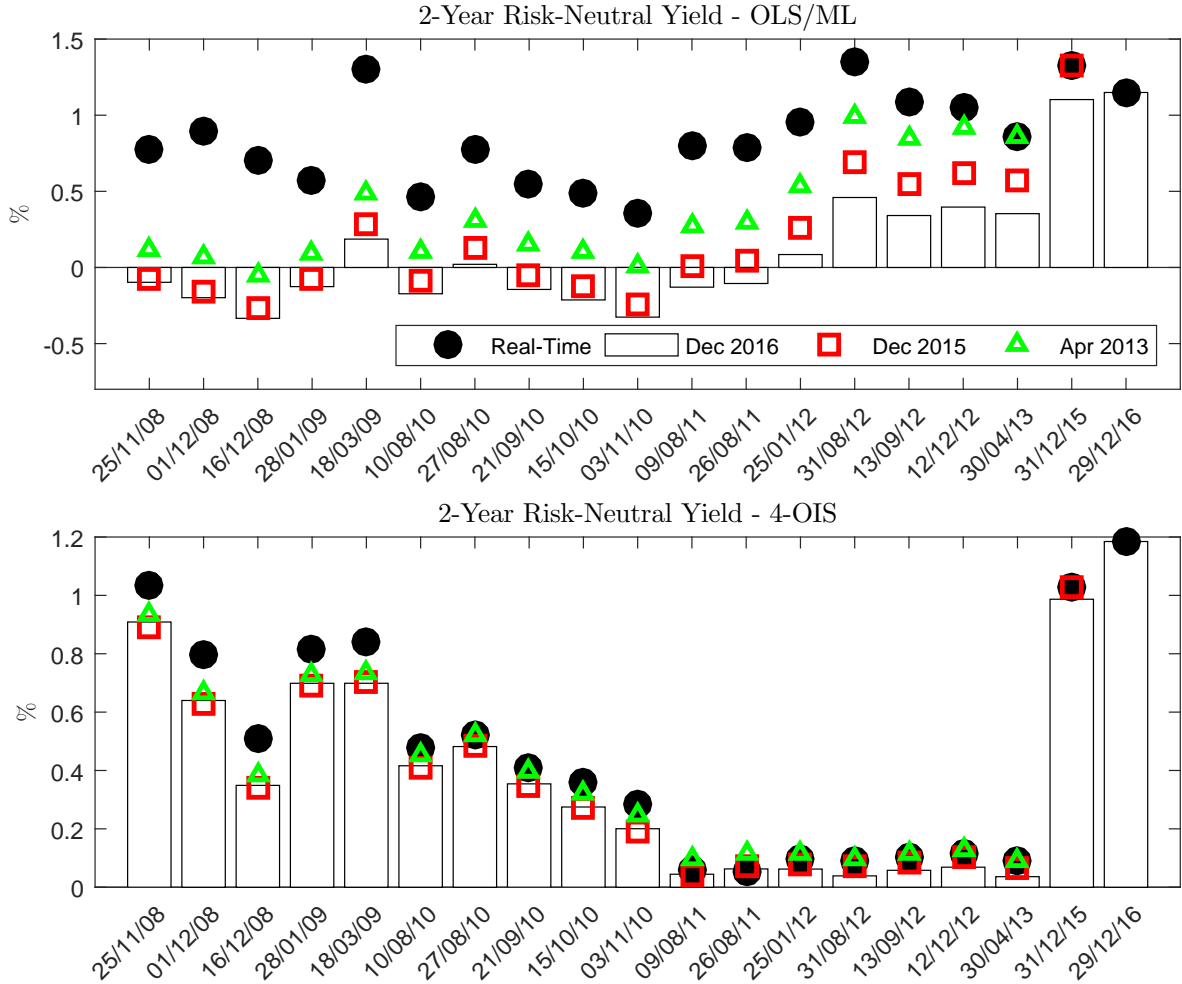
⁵⁵The 16 dates are: 25/11/2008; 01/12/2008; 16/12/2008; 28/01/2009; 18/03/2009; 10/08/2010; 27/08/2010; 21/09/2010; 15/10/2010; 03/11/2010; 09/08/2011; 26/08/2011; 25/01/2012; 31/08/2012; 13/09/2012; and 12/12/2012.

Figure 10: Estimates of the 2-Year Fitted Yield from the Unaugmented OLS/ML and 4-OIS-Augmented GADTSMs on 19 Different Dates



Note: Estimates of the 2-year fitted yield from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on 19 dates, 16 of which are associated with US unconventional monetary policy announcements in [Lloyd \(2017c\)](#). All samples begin in January 2002. The black dots represent real-time estimates of bond yields, using data up to the event date. The white bars represent estimates using a sample that ends in December 2016, the red squares represent estimates using a sample that ends in December 2015, and the green triangles represent estimates using a sample that ends in April 2013. All models are estimated with three pricing factors, using daily data. Yields are plotted in annualised percentage points. The date format for events, on the horizontal axis, is DD/MM/YY.

Figure 11: Estimates of the 2-Year Risk-Neutral Yield from the Unaugmented OLS/ML and 4-OIS-Augmented GADTSMs on 19 Different Dates



Note: Estimates of the 2-year risk-neutral yield from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on 19 dates, 16 of which are associated with US unconventional monetary policy announcements in [Lloyd \(2017c\)](#). All samples begin in January 2002. The black dots represent real-time estimates of risk-neutral yields, using data up to the event date. The white bars represent estimates using a sample that ends in December 2016, the red squares represent estimates using a sample that ends in December 2015, and the green triangles represent estimates using a sample that ends in April 2013. All models are estimated with three pricing factors, using daily data. Yields are plotted in annualised percentage points. The date format for events, on the horizontal axis, is DD/MM/YY.

close to real-time and longer-sample estimates on other event days. For instance, on event day 6 (August 10, 2010), the real-time estimate of the OIS-augmented 2-year yield is 48 basis points, the estimate from the 2002-2015 sample is 41 basis points, while an estimate using a 6.5-year sample from January 2004 to the event date is 45 basis points. Therefore, inference about interest rate expectations can be reliably made from the OIS-augmented model, regardless of the sample period chosen.

F.2 Four-Factor Specification

In the light of evidence by [Cochrane and Piazzesi \(2005, 2008\)](#) and [Duffee \(2011\)](#), who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields, I estimate a four-factor specification of the OLS/ML, bias-corrected, survey-augmented and 4-OIS-augmented GADTSMs. Although the four-factor model better fits actual bond yields for the 2002-2016 sample, I do not present these results in the main body of the paper because the economic meaning of the pricing factors in a three-factor model is well understood (i.e., level, slope and curvature), while the economic interpretation the fourth factor is less well understood. Moreover, the differences in risk-neutral yield estimates from the three and four-factor models are small, and there is no evidence that a single model is unambiguously preferable.

I estimate the four-factor model using the same underlying daily data as the three-factor model presented in the main body of the paper.

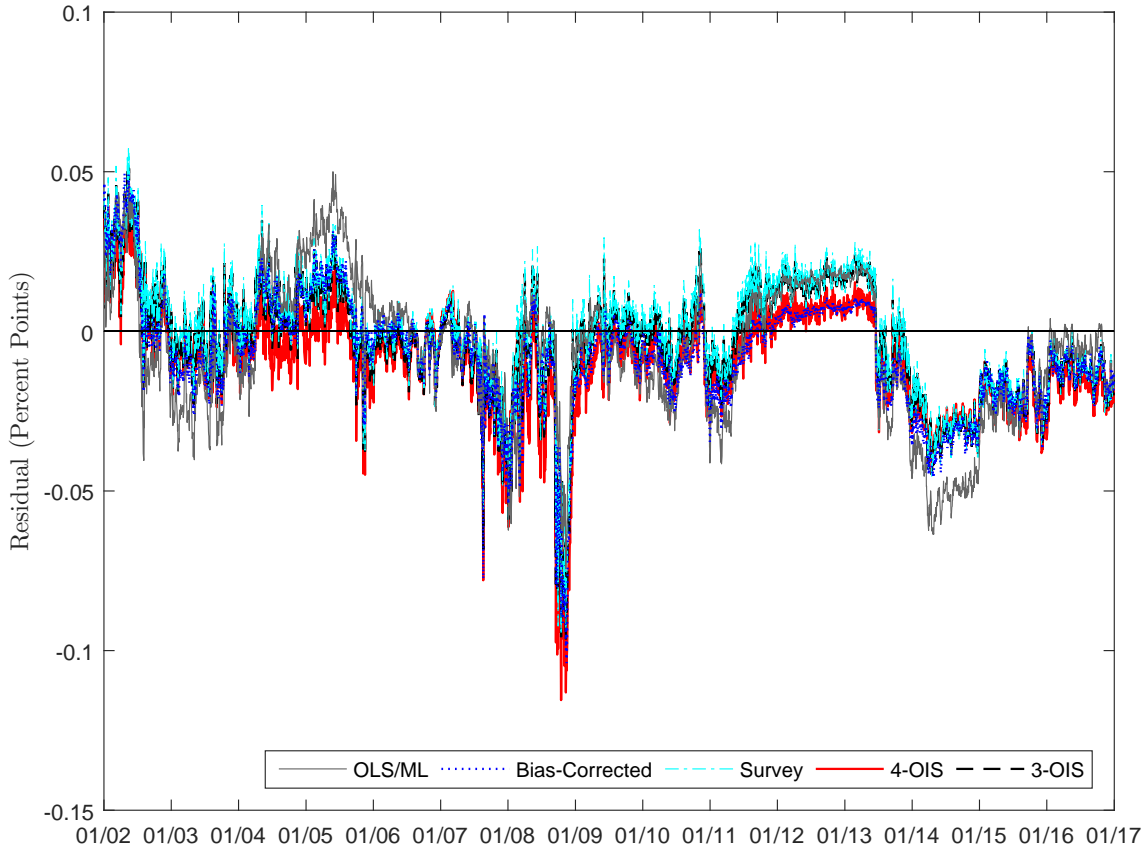
Fitted Yields Figure 12 demonstrates that the fitted yields from the four-factor GADTSMs do not differ markedly from one another. Here I plot the residual of the 2-year fitted yield from the four-factor model. The average RMSE for each of the models at all thirteen maturities is around 3 basis points, around 2 basis points smaller than from the three-factor model.

Fitted OIS Rates As with the three-factor models, the four-factor OIS-augmented models accurately fit OIS rates. Figure 13 demonstrates this, plotting the actual and fitted 3, 6, 12 and 24-month OIS rates. Table 10 presents the RMSE for the fitted OIS rates from each of the OIS-augmented models, comparing the results from the three and four-factor models. The table illustrates that the three-factor OIS-augmented models perform marginally better than their four-factor counterparts on a RMSE basis.

Pricing Factors Figure 14 plots the first four estimated principal components of the daily frequency bond yield data, and the four estimated pricing factors from the 4-OIS-augmented model. As with the three-factor model, the plot demonstrates that the inclusion of OIS rates in the estimation of GADTSMs does not significantly influence the bond pricing factors, as the quantities closely co-move.

Interest Rate Expectations Figure 15 plots the 3 and 6-month ahead 1-month risk-neutral forward yields from the four-factor models against comparable-horizon federal funds futures rates. The figure demonstrates that the OIS-augmented models provide the closest fit for

Figure 12: Residual of the 2-Year Fitted Yield from Four-Factor GADTSMs



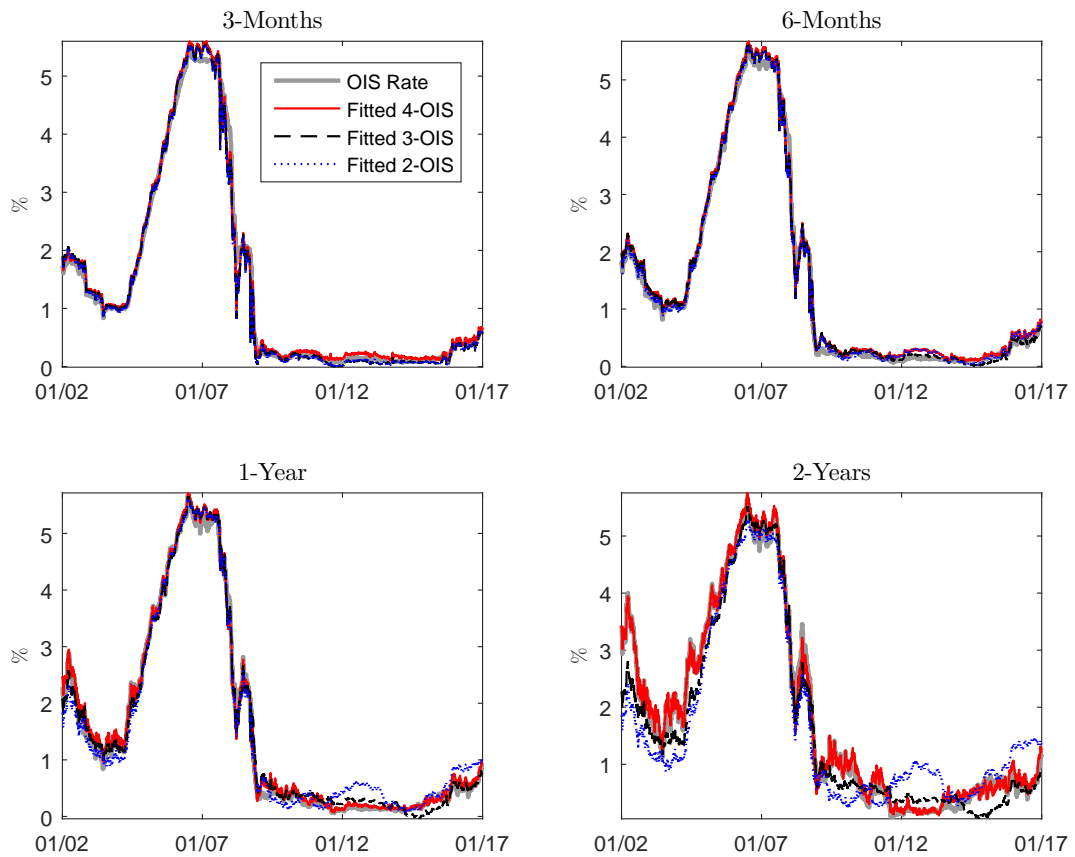
Note: Residuals of the 2-year fitted yield from five monthly frequency GADTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model; and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2016. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

Table 10: GADTSM Fit: Root Mean Square Error (RMSE) of Fitted OIS Rates *vis-à-vis* the Actual OIS Rates for Three and Four-Factor Models

Sample: January 2002 to December 2016						
Maturity	Three-Factor Model			Four-Factor Model		
	2-OIS	3-OIS	4-OIS	2-OIS	3-OIS	4-OIS
3-Months	0.1183	0.1206	0.1305	0.1565	0.1561	0.1414
6-Months	0.0924	0.1088	0.0861	0.1136	0.1113	0.1219
1-Year	0.1585	0.0926	0.0831	0.2215	0.1324	0.0932
2-Year	0.5306	0.2634	0.0985	0.5710	0.3501	0.1091

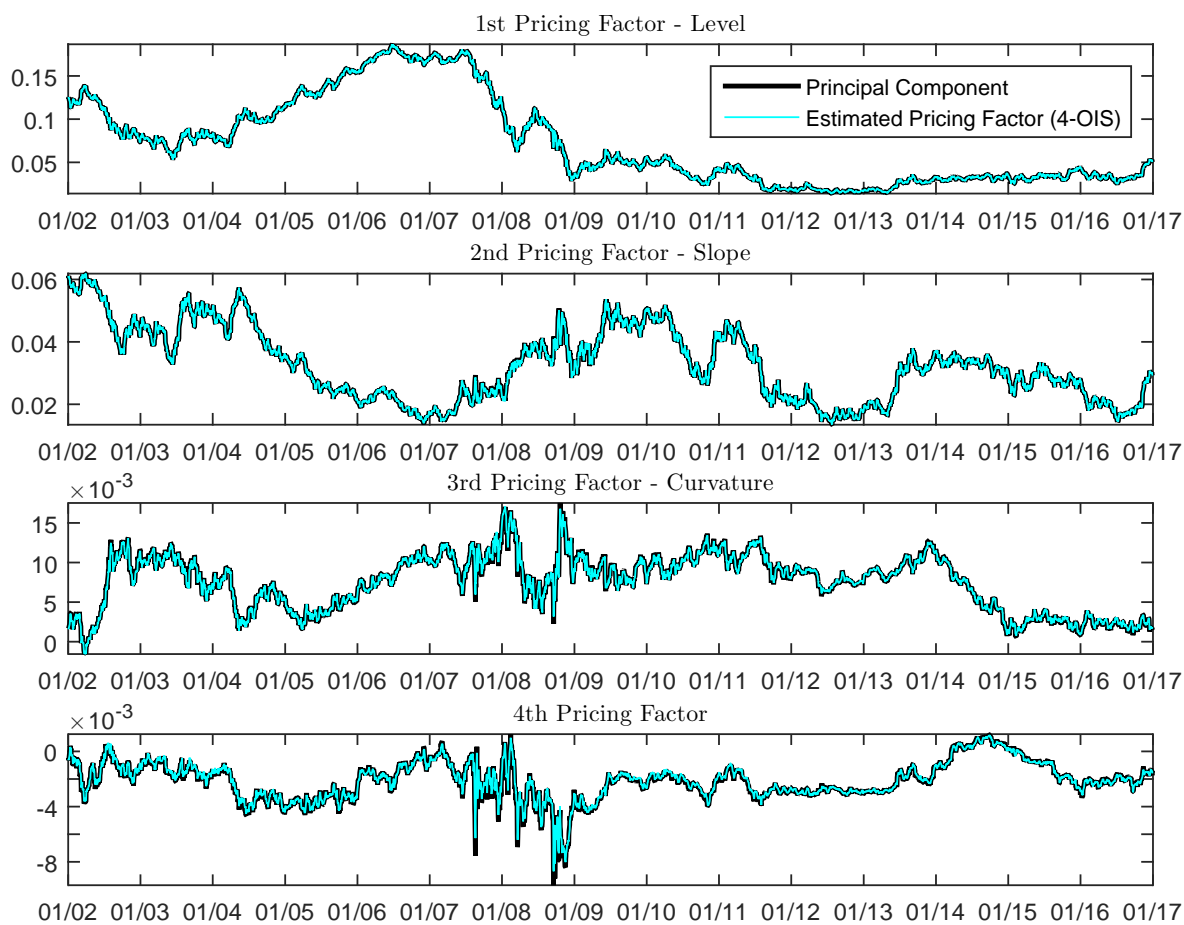
Note: RMSE of the fitted OIS rates from each of the three and four-factor OIS-augmented GADTSMs, computed by comparing the model-implied fitted OIS rate to the actual OIS rate on each day. All figures are expressed in annualised percentage points. The three GADTSMs are: (i) the 2-OIS-augmented model (2-OIS); (ii) the 3-OIS-augmented model (3-OIS); and (iii) the 4-OIS-augmented model (4-OIS). The lowest RMSE model at each maturity has been emboldened for ease of reading.

Figure 13: Fitted OIS Rates from the Four-Factor OIS-Augmented Models



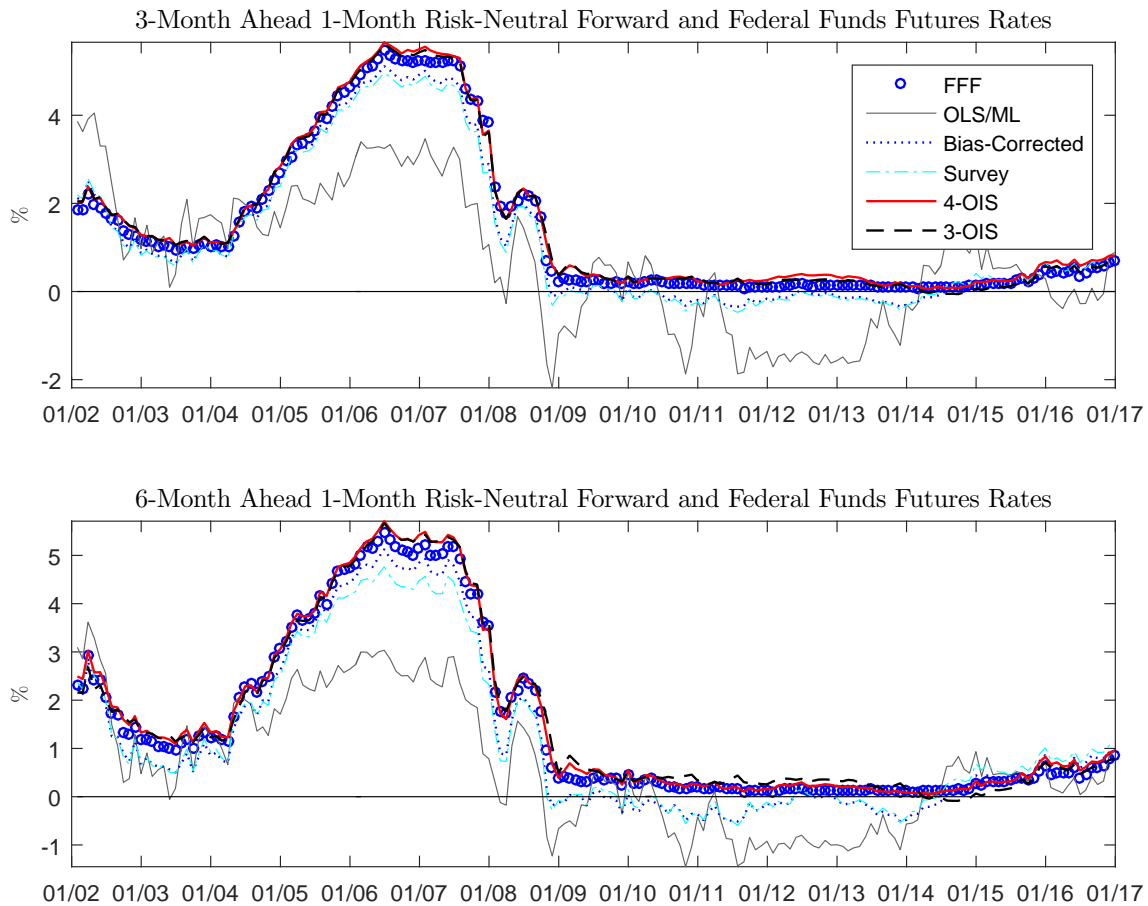
Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with four pricing factors using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

Figure 14: First Four Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the Four-Factor 4-OIS-Augmented Model



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the four-factor 4-OIS-augmented model implied by the Kalman filter.

Figure 15: Estimated Risk-Neutral 1-Month Forward Yields from the Four-Factor Models and Comparable-Horizon Federal Funds Futures (FFF) Rates



Note: I plot estimated 3 to 4-month ahead and 6 to 7-month ahead risk-neutral forward yields from each of five GADTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates. All figures are in annualised percentage points.

market-based measures of interest rate expectations for the majority of the 2002-2016 sample. Moreover, the risk-neutral forward yields from the four-factor OIS-augmented models are similar to those from their three-factor variants plotted in figure 6. In contrast, the four-factor OLS/ML performs visibly worse than its three-factor counterpart.

Table 11 presents a RMSE comparison of the risk-neutral 1-month forward yields from the four-factor models. For comparison, the rightmost column of the table presents the corresponding RMSE from the three-factor 4-OIS-augmented model. The table demonstrates that, of all four-factor models, the OIS-augmented models continue provide the best estimates of interest rate expectations on a RMSE basis. However, a comparison of the four and three-factor models indicates that neither unambiguously outperforms the other. The three-factor 4-OIS-augmented model provides the lowest RMSE fit for 9 of the 11 forward yields.

Table 11: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the Risk-Neutral 1-Month Forward Yields *vis-à-vis* Corresponding-Horizon Federal Funds Futures Rates for Four-Factor Models in Comparison to the Three-Factor 4-OIS-Augmented Model

Sample: January 2002 to December 2016						
Horizon	Four-Factor Model					Three-Factor Model
	OLS/ML	BC	Survey	3-OIS	4-OIS	4-OIS
0 to 1 Months	1.9394	0.2871	0.3196	0.2406	0.1923	0.1823
1 to 2 Months	1.5617	0.2647	0.3164	0.1443	0.1384	0.1293
2 to 3 Months	1.4007	0.2785	0.3395	0.1209	0.1438	0.0929
3 to 4 Months	1.3265	0.3037	0.3738	0.1302	0.1515	0.0828
4 to 5 Months	1.2813	0.3287	0.4065	0.1436	0.1491	0.0898
5 to 6 Months	1.2497	0.3573	0.4391	0.1565	0.1401	0.1021
6 to 7 Months	1.2324	0.3937	0.4746	0.1721	0.1320	0.1176
7 to 8 Months	1.2253	0.4354	0.5096	0.1924	0.1295	0.1316
8 to 9 Months	1.2338	0.4857	0.5502	0.2151	0.1304	0.1429
9 to 10 Months	1.4861	0.9388	0.9627	0.7346	0.6630	0.6564
10 to 11 Months	1.7709	1.3067	1.3164	1.0411	0.9703	0.9635

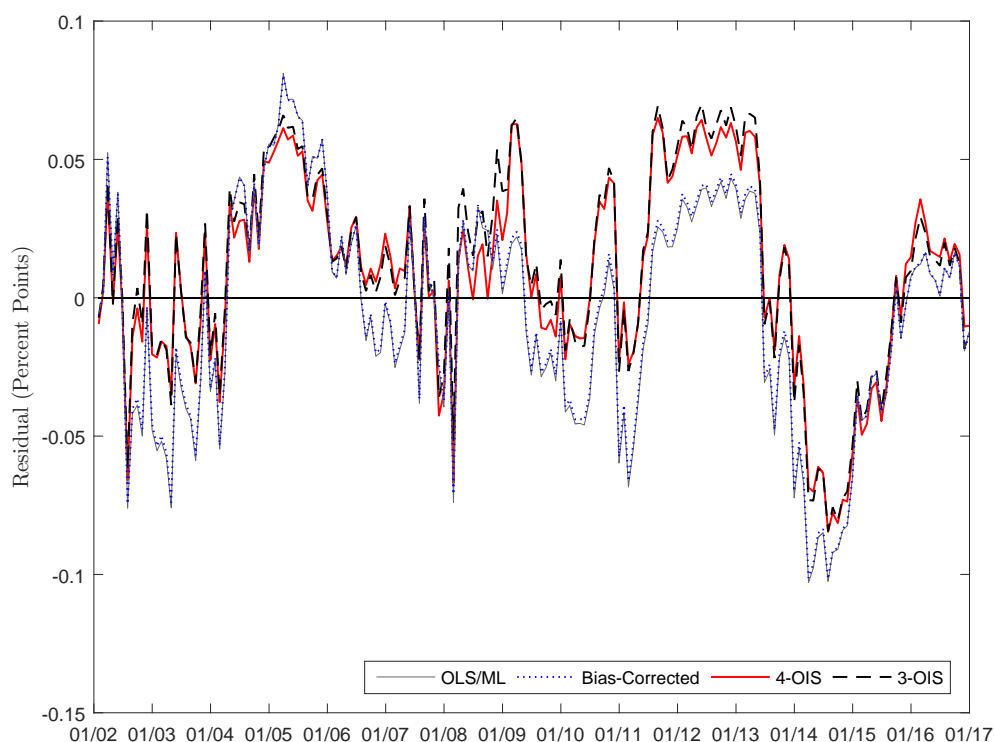
Note: RMSE of the risk-neutral 1-month forward yields from six GADTSMs in comparison to corresponding-horizon federal funds futures rates. The six models are: (i) the unaugmented four-factor model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected four-factor model (Bias-Corrected); (iii) the survey-augmented four-factor model (Survey); (iv) the 3-OIS-augmented four-factor model (3-OIS); (v) the 4-OIS-augmented four-factor model (4-OIS); and (vi) the 4-OIS-augmented three-factor model (4-OIS) from the main body of the paper. The models are estimated using daily data from January 2002 to December 2016. The risk-neutral forward yields and the federal funds futures rates are compared on the final day of each calendar month. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

F.3 Monthly Frequency GADTSMs

For robustness, I also estimate the GADTSMs at a monthly frequency. The monthly frequency models have the same structure as described in the main body of the paper, with the time index t now representing a month, rather than a day. To estimate the model, I use bond yields and OIS rates from the final day of each calendar month. I estimate the monthly frequency models using the same 13 bond yields and 4 OIS rates for the January 2002 to December 2016. The headline conclusion is as follows: the benefits of OIS-augmentation for estimates of future short-term interest rate expectations carry over from daily frequency estimation to lower frequencies, such as the monthly frequency.

Fitted Yields Figure 16 illustrates that the fitted yields from the monthly frequency GADTSMs do not differ markedly (i) from one another and (ii) in comparison to the daily frequency estimates presented in the main body of the paper. Here, I plot the residual of the 2-year fitted yield from the monthly frequency GADTSMs. They serve to illustrate that the models provide a similar fit for actual bond yields.

Figure 16: Residual of the 2-Year Fitted Yield from Monthly Frequency GADTSMs



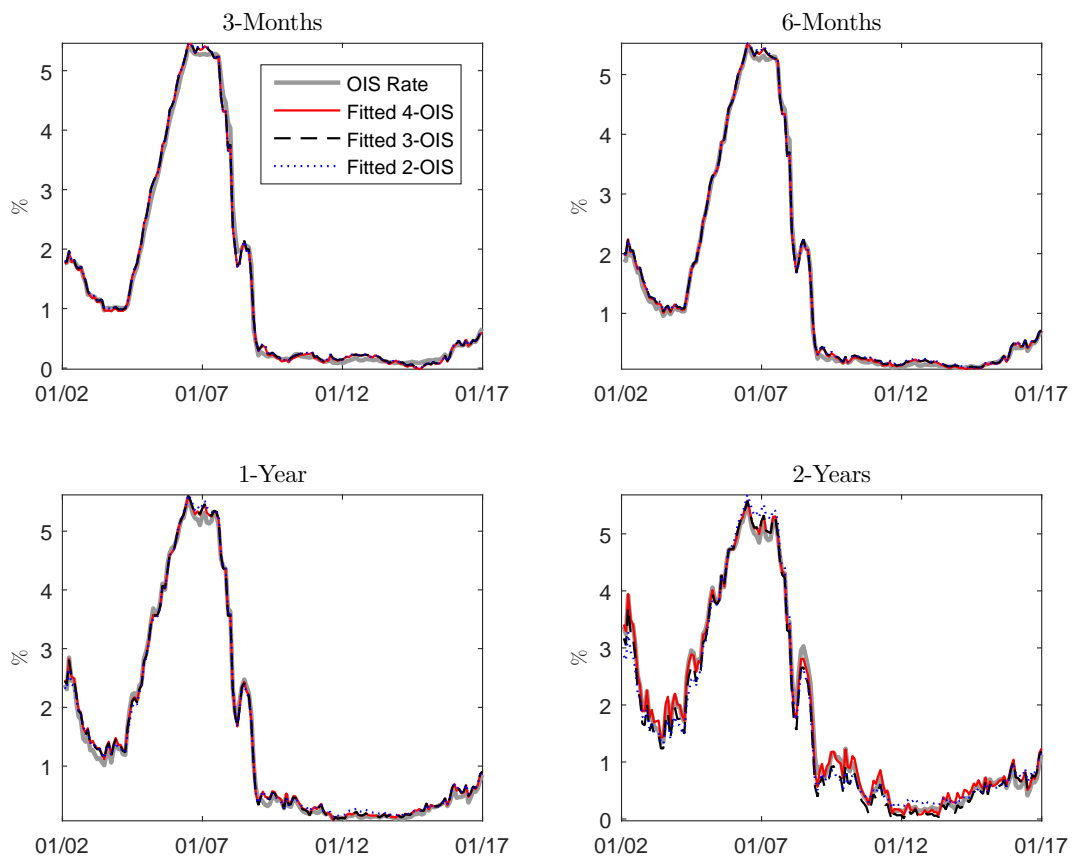
Note: Residuals of the 2-year fitted yield from four monthly frequency GADTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model; and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2016. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

Fitted OIS Rates As with the daily frequency results, the monthly frequency OIS-augmented models accurately fit OIS rates. Figure 17 demonstrates, again, that the 4-OIS-augmented model accurately fits the 3, 6, 12 and 24-month OIS rates. Although the 4-OIS-augmented provides a visually superior fit of all four OIS rates, the 2 and 3-OIS-augmented models do provide estimates of OIS rates that fit actual OIS rates reasonably well.

Pricing Factors Figure 18 plots the estimated principal components of the monthly frequency bond yield data and the estimated pricing factors from the monthly-frequency 4-OIS-augmented model. As with the daily frequency model, the plot demonstrates that the inclusion of OIS rates in the estimation of GADTSMs does not significantly influence the bond pricing factors. The two quantities evolve almost identically.

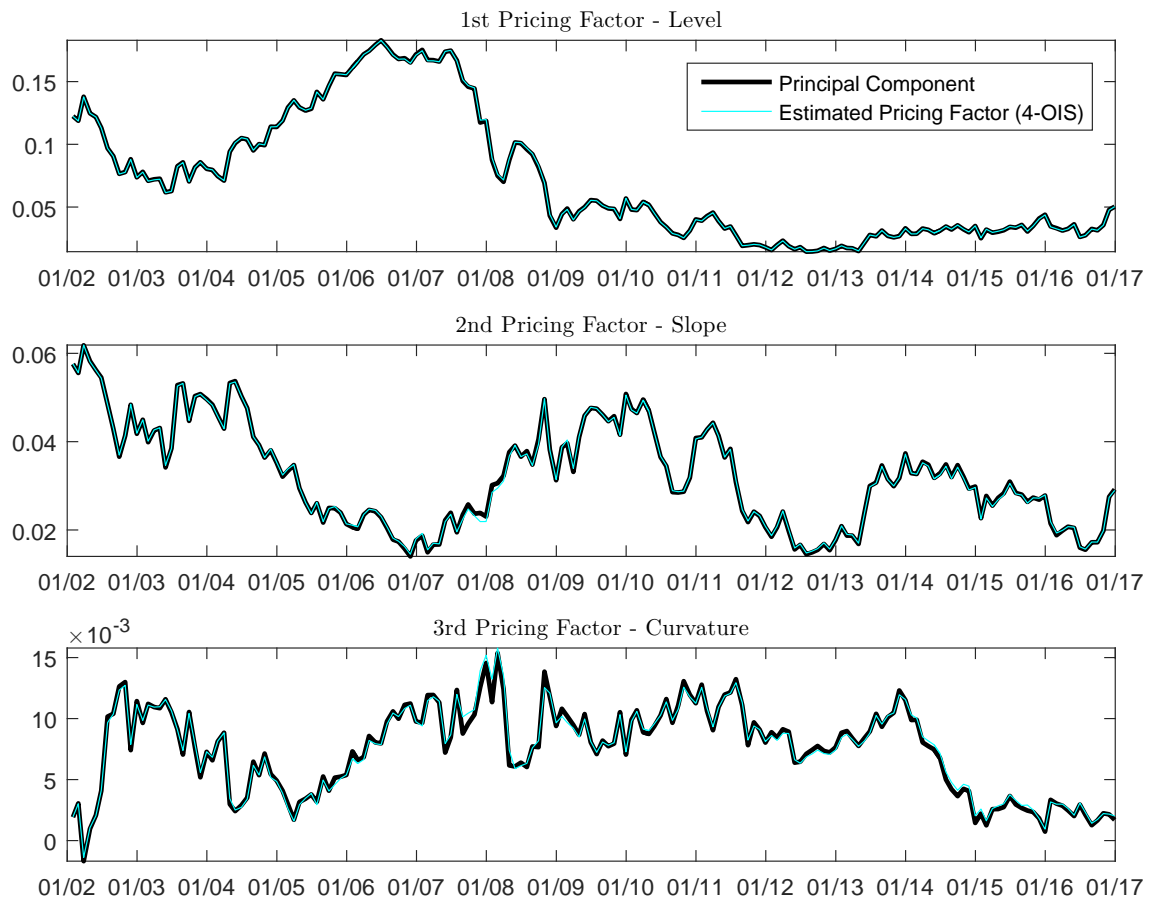
Interest Rate Expectations Finally, in figure 19, I plot the 6-month and 1-year risk-neutral yields from the monthly frequency OLS/ML, bias-corrected, 4 and 3-OIS-augmented GADTSMs. The figure highlights that the monthly frequency estimates for the level of interest rate expectations at a given time are similar, qualitatively and quantitatively, to estimates using daily frequency data.

Figure 17: Fitted OIS Rates from the Monthly Frequency OIS-Augmented Models



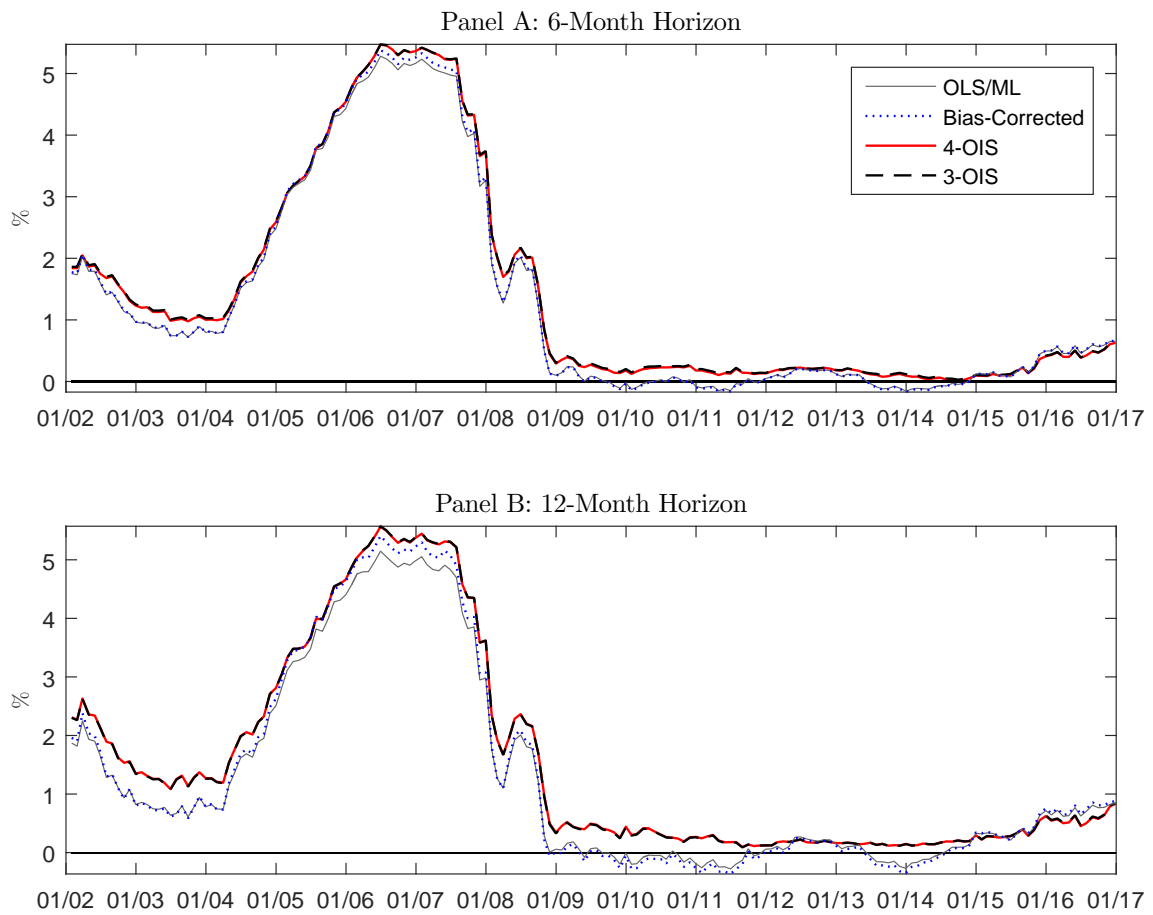
Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with three pricing factors using end of month data from January 2002 to December 2016. All figures are in annualised percentage points.

Figure 18: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model at a Monthly Frequency



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model implied by the Kalman filter.

Figure 19: Short-Term Interest Rate Expectations from the Monthly Frequency Models



Note: I plot estimated 6-month and 1-year risk-neutral yields from each of four GADTSMs in panels A and B, respectively. The four models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model (4-OIS); and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2016. All figures are in annualised percentage points.

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