Decomposing and backtesting a flexible specification for CoVaR

Giovanni Bonaccolto*

Massimiliano Caporin†

Sandra Paterlini‡

This version: November 26, 2017

Abstract

The Conditional Value-at-Risk (CoVaR) proposed by Adrian and Brunnermeier (2016)—which quantifies the impact of a company in distress on the Value-at-Risk (VaR) of the financial system—has established itself as a reference measure of systemic risk. In this study, we extend the CoVaR along two dimensions, which lead respectively to: i) the Conditional Autoregressive VaR (CoCaViaR), in which we include autoregressive components of conditional quantiles to explicitly capture volatility clustering and heteroskedasticity; ii) the Conditional Quantile-Located VaR (QL-CoVaR), which accentuates the degree of distress in the connections between the conditioning companies and the financial system, as the parameters are estimated by directly linking the left tails of their returns' distributions. By combining the two new risk measures, we also build the Conditional Autoregressive Quantile-Located VaR (QL-CoCaViaR) and introduce a new backtesting method. A large empirical analysis highlights the validity of such approaches and critically discuss their pros and cons. In particular, including quantile-located relationships leads to relevant improvements in terms of predictive accuracy during stressed periods and, therefore, provides a valuable tool for regulators to assess systemic events.

1 Introduction

Recent financial crises have highlighted the need to develop better tools and measures to quantify and predict systemic risk, emphasizing the importance of the interconnectedness of firms, their exposure to systemic events as well as the marginal effect each company has on the entire system. The collapse of Lehman Brothers and the aftermath of the subprime crisis have prompted regulators to move beyond

*University of Enna "Kore", Italy. Email: giovanni.bonaccolto@unikore.it

†Department of Statistical Sciences, University of Padova, Italy. Email: massimiliano.caporin@unipd.it

‡Corresponding author. Department of Finance and Accounting, European Business School, Germany. Email: sandra.paterlini@ebs.edu

the unconditional Value-at-Risk (VaR), which does not capture the dependence structure of extreme co-movements in stock markets and the consequent spillover effects generated during stressed phases.

Adrian and Brunnermeier (2016) moved away from unconditional VaR estimates, introducing the Conditional Value-at-Risk (CoVaR), that has become one of most important measures for systemic risk. In contrast to the VaR, which only quantifies the individual tail-risk of the system (or a company), the CoVaR takes into account the impact of a company in distress on the entire system. The CoVaR of the system is estimated using a quantile regression model (Koenker and Bassett, 1978) where one of the covariates is the return of a conditioning company ($x_{i,t}$, for $i = 1, ..., N$). Then, after estimating this model and replacing $x_{i,t}$ with its τ-th quantile—a low quantile representing the company's VaR as the focus is on left-tail relations, e.g., $\tau = \{0.01, 0.05, 0.10\}$—it is possible to measure the VaR of the system conditional to the stressed state of the i-th firm. Similarly, it is possible to measure the VaR of the system conditional to the median or normal state of the i-th company—by replacing $x_{i,t}$ with its 0.5-th quantile in the estimated model—and the difference between the two estimates leads to the ΔCoVaR, which quantifies the marginal contribution of an individual company to the systemic risk (Adrian and Brunnermeier, 2016).

By relying on quantile regression, Adrian and Brunnermeier (2016) introduced a simple but yet informative risk measure. As a result, the CoVaR has attracted much attention and has been applied in many empirical studies. For instance, López-Espinosa et al. (2012) used the CoVaR to identify the drivers of large international banks, which are significantly related to systemic risk. Bernal et al. (2014) studied the contribution of the banking, insurance and other financial services sectors to systemic risk using the ΔCoVaR. Castro and Ferrari (2014) proposed a significance test for the ΔCoVaR to identify/rank systemically important companies. Bernardi et al. (2015) developed a Bayesian inference for CoVaR estimation. López-Espinosa et al. (2015) extended the CoVaR measure to study the asymmetric response of the banking system to positive and negative returns of individual banks. Bernardi et al. (2017) generalized the CoVaR in a multiple quantile setting.

Here, we extend the CoVaR into two main dimensions. First, we take into account the fact that the volatility—and then the distribution—of financial returns changes over time. Nevertheless, the CoVaR method does not include any factor reflecting the heteroskedastic behavior of financial returns. We bridge this gap by adding autoregressive components capturing the quantiles' dynamics over time, inspired by the Conditional Autoregressive Value-at-Risk (CaViaR) introduced by Engle and Manganelli (2004). The heteroskedasticity in the CoVaR's dynamics is also considered—with a different approach based on GARCH modelling—by Girardi and Ergun (2013). In particular, it is interesting to check whether and in what measure the inclusion of the CaViaR's components—which capture the persistence of quantiles over time and link the current quantile of a financial return with lagged values of the same return—affect the relations between the financial system and the conditioning companies.

Second, the CoVaR's parameters are estimated conditional to the return of the i-th company ($x_{i,t}$)

and not to the quantile of $x_{i,t}$. Nevertheless, correlations between financial institutions increase during stressed periods and the risk of contagion threatens the stability of the entire economy due to potential spillover effects. Consequently, the information content of extreme risk measures becomes increasingly relevant when accentuating the distress degree in financial connections. Here, we emphasize the impact exerted by a conditioning company in the neighbourhood of its τ-th quantile on the VaR of y_t—the return of the system—and, thus, increase the distress degree in the relations between this company and the entire system by directly linking the left tails of their returns' distributions. In other words, we use an estimation process which reflects the joint stressed state of the system and of the i-th firm. We define the resulting measure as Conditional Quantile-Located Value-at-Risk (QL-CoVaR). In particular, we estimate the QL-CoVaR's parameters with a weighted quantile regression model building on kernel-based weights, following the approach that Sim and Zhou (2015) used to study the relations between oil prices and stock returns. To the best of our knowledge, such a method has never tested within a CoVaR framework for systemic risk.

By combining the CoCaViaR and the QL-CoVaR, we also introduce the Conditional Autoregressive Quantile-Located Value-at-Risk (QL-CoCaViaR), where the conditional VaR depends on both an autoregressive component and a quantile of the independent variable. Building on the QL-CoCaViaR model, which nests the two previously introduced specifications, we also introduce a further innovative element, that is, a decomposition of the resulting ΔQL-CoCaViaR, providing an economic interpretation for each component and studying their contribution to the overall risk measure. Therefore, we can analyze how the CaViaR and the quantile-located effects interact when combined in the same model —the QL-CoCaViaR—and whether this combination is better than considering the individual components separately—in the CoCaViaR and in the QL-CoVaR, respectively.

The three risk measures we propose here are not the only extensions of the CoVaR in the literature. To the best of our knowledge, the contributions that most closely relate to ours are White et al. (2015) and Girardi and Ergun (2013). In particular, White et al. (2015) proposed the 'VAR for VaR'—a multivariate version of the CaViaR model introduced by Engle and Manganelli (2004). Similarly to our approach, the 'VAR for VaR' captures both the persistence of the system's quantiles and the relations between the system's and the i-th company's lower quantiles. Nevertheless, our approach differs in three important points. First, the 'VAR for VaR' links the past of a single company ($x_{i,t-1}$) to the present of the system (y_t), whereas we focus on contemporary relationships to asses how the system immediately reacts when an institution enters a financial distress. Second, in White et al. (2015), the system's and the i-th company's quantiles are estimated in a bivariate setting in which the covariates are their respective latent (lagged) values. In contrast, the return of the i-th company we include in the equation of the system's quantile is not latent, being observed. As a result, the impact of the i-th company on the system is estimated using a different approach. Third, in White et al. (2015), the financial distress of the conditioning company occurs when its return is exactly at its VaR. Similarly to Girardi and Ergun (2013), we provide more flexibility as the financial distress of the conditioning company does not necessarily occur when $x_{i,t}$ is exactly at its VaR. In particular, in Girardi and Ergun (2013), the financial institution can be at most at its VaR to be in a stressed state. In contrast, we are aware that the distress state of the i-th company might not be confined to the case of observing returns lower than or equal to a given quantile. In fact, we focus on the impact exerted by $x_{i,t}$ in the (left and right) neighbourhood of its τ-th quantile, increasing the flexibility and smoothing discontinuities in the identification of the company distress.

We compare the competitive risk measures both in- and out-of-sample, using a large dataset including more than 1,000 banks and insurance companies. In particular, we checked that the relations between the system and the individual companies become stronger when considering the quantile-located effects, that is, when accentuating the distress degree in their connections. In fact, the distance between the quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—and the other indicators—ΔCoVaR and ΔCoCaViaR—is relatively low during calm periods and becomes accentuated during stressed phases—e.g., during the subprime crisis, during the 'internet bubble' in 2000, after the terrorist attacks in September 2001, around the stock market crash in 2002, during the war in Iraq (years 2001—2003) and during the European sovereign debt crisis (years 2010—2011). We then highlight the capability of the quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—to react more strongly during systemic events. The CaViaR's components absorb part of the relations between the system and the conditioning companies, because of the persistence of the y_t's quantiles over time. As a result, the CaViaR is more restrictive than the CoVaR and the QL-CoCaViaR is more restrictive than the QL-CoVaR.

We also evaluate the out-of-sample performance of the risk measures using various backtesting methods. However, well-known backtesting approaches are, in our opinion, not appropriate for quantile-located risk measures. Therefore, as a further contribution, we introduce novel backtesting methods tailored to the quantile-located risk measures we use here. Our analysis shows that the quantile-located relationships are particularly useful in improving the predictive accuracy during stressed periods, with the QL-CoVaR outperforming the other competitive measures.

The work is structured as follows. We present the risk measures and the backtesting methods in Section 2. Section 3 describes the dataset, whereas Section 4 reports the empirical results. Section 5 concludes.

2 Notation and methods

Let y_t and $x_{i,t}$ be the returns of the financial system and of the i-th financial company at time t, respectively, for $i = 1, ..., N$ and $t = 1, ..., T$, whereas $\mathbf{M}_t$ is a $k-$dimensional row vector including a set of control variables observed at time t. Let $Q_\tau(x_{i,t}|\mathbf{I}_{t-1})$ denotes the τ-th quantile of $x_{i,t}$, for $\tau \in (0, 1)$, conditional to the information set $\mathbf{I}_{t-1}$, where $\mathbf{I}_{t-1} = (y_{t-1}, x_{i,t-1}, \mathbf{M}_{t-1})$. Similarly, $Q_\theta(y_t|\mathbf{I}_{t-1}, x_{i,t})$ is the θ-th quantile of y_t conditional to the information set available at $t-1$ as well as to the return of the i-th company observed at time t, for $\theta \in (0,1)$. For simplicity, we set $Q_\tau(x_{i,t}|\mathbf{I}_{t-1}) \equiv Q_\tau(x_{i,t})$ and $Q_\theta(y_t|\mathbf{I}_{t-1}, x_{i,t}) \equiv Q_\theta^{(i)}(y_t)$. As we focus on the left-tail dependence between y_t and $x_{i,t}$, θ and τ take low values, typically in the interval $(0, 0.05)$. Hence, $Q_\theta^{(i)}(y_t)$ and $Q_\tau(x_{i,t})$ are interpreted as the Values-at-Risk (VaR), at the levels θ and τ, of the financial system and of the i-th company, respectively.

2.1 Conditional Value-at-Risk

Adrian and Brunnermeier (2016) introduced the Conditional Value-at-Risk (CoVaR) as a measure of systemic risk. The CoVaR builds on the estimation of the following (linear) conditional quantile models:

$$Q_\tau(x_{i,t}) = \alpha_\tau^{(i)} + \boldsymbol{\beta}_\tau^{(i)} \mathbf{M}'_{t-1}, \tag{1}$$

$$Q_\theta^{(i)}(y_t) = \delta_\theta^{(i)} + \lambda_\theta^{(i)} x_{i,t} + \boldsymbol{\gamma}_\theta^{(i)} \mathbf{M}'_{t-1}, \tag{2}$$

where $\alpha_\tau^{(i)}$, $\delta_\theta^{(i)}$, and $\lambda_\theta^{(i)}$ are scalars, while $\boldsymbol{\beta}_\tau^{(i)}$ and $\boldsymbol{\gamma}_\theta^{(i)}$ are k-dimensional row vectors of parameters. The subscripts of the parameters in (1)—(2) point out their dependence on the quantiles levels τ and θ, respectively; in contrast, the superscript (i) indicates that (1)—(2) are specific to the i-th company, for $i = 1, ..., N$. Although the intercept $\delta_\theta^{(i)}$ and the vector $\boldsymbol{\gamma}_\theta^{(i)}$ in (2) are not directly linked to the i-th company, they might be affected by the relations between y_t and $x_{i,t}$. Therefore, $\delta_\theta^{(i)}$ and $\boldsymbol{\gamma}_\theta^{(i)}$ in (2) are also indexed by (i).

The parameters in (1)—(2) are estimated through the quantile regression method introduced by Koenker and Bassett (1978). Their standard errors could be computed following various approaches. Here, we use a bootstrap method (Efron, 1979), that is, the xy-pair approach of Kocherginsky (2003), that provides accurate results without any distributional assumption. After obtaining the estimated quantile $\widehat{Q}_\tau(x_{i,\tau}) = \widehat{\alpha}_\tau^{(i)} + \widehat{\boldsymbol{\beta}}_\tau^{(i)} \mathbf{M}'_{t-1}$, the CoVaR of the financial system, conditional to the VaR of the i-th company, is computed as follows:

$$CoVaR_{t,\theta,\tau}^{(i)} = \widehat{\delta}_\theta^{(i)} + \widehat{\lambda}_\theta^{(i)} \widehat{Q}_\tau(x_{i,t}) + \widehat{\boldsymbol{\gamma}}_\theta^{(i)} \mathbf{M}'_{t-1}. \tag{3}$$

For the sake of brevity, we do not use t, θ and τ as subscripts, as well as (i) as superscript, when we refer to the CoVaR as a risk measure throughout the paper. Hence, the CoVaR represents the risk of the financial system at the θ level (as in the case of the traditional VaR) conditional to the fact that the i-th financial company is in a stressed state, that is, its return is equal to its Value-at-Risk: $x_{i,t} = \widehat{Q}_\tau(x_{i,\tau})$. Note that $Q_\tau(x_{i,t})$ and $Q_\theta^{(i)}(y_t)$ in (1)—(2) depend on a common set of control variables, that is, the ones included in $\mathbf{M}'_{t-1}$.

The CoVaR can be also computed conditional to the normal (or median) state of the i-th company, such that:

$$CoVaR^{(i)}_{t,\theta,1/2} = \widehat{\delta}^{(i)}_{\theta} + \widehat{\lambda}^{(i)}_{\theta} \widehat{Q}_{1/2}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta} \mathbf{M}'_{t-1}. \tag{4}$$

We highlight that the quantile level θ does not change between (3) and (4). Indeed, the coefficients in (3)—(4) are identical, being estimated from the same quantile regression model in (2). Then, by subtracting $CoVaR^{(i)}_{t,\theta,1/2}$ from $CoVaR^{(i)}_{t,\theta,\tau}$, we compute the so-called ΔCoVaR (Adrian and Brunnermeier, 2016) to quantify the marginal contribution of the i-th company to the systemic risk. In fact, the ΔCoVaR captures the system reaction to the deterioration of the i-th financial company risk, as quantified by the difference between the median return and the Value-at-Risk of the company (both obtained from a quantile regression approach). Given that $CoVaR^{(i)}_{t,\theta,1/2}$ is always parameterized to the median state of the i-th conditioning company, we can omit the level 1/2 as subscript of the ΔCoVaR measure as follows:

$$\Delta CoVaR^{(i)}_{t,\theta,\tau} = CoVaR^{(i)}_{t,\theta,\tau} - CoVaR^{(i)}_{t,\theta,1/2} = \widehat{\lambda}^{(i)}_{\theta} \left[\widehat{Q}_{\tau}(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t}) \right]. \tag{5}$$

For simplicity, we estimate the lower quantiles of y_t and $x_{i,t}$ (i.e. $\widehat{Q}^{(i)}_{\theta}(y_t)$ and $\widehat{Q}_{\tau}(x_{i,t})$) at the same quantile level: $\theta = \tau$. Hence, we can further simplify the notation by setting $\Delta CoVaR^{(i)}_{t,\theta,\tau} \equiv \Delta CoVaR^{(i)}_{t,\tau}$.

2.2 Conditional Autoregressive Value-at-Risk

Financial returns exhibit volatility clustering, especially during crisis periods (Cont, 2001).[1] Volatility changes over time, affecting the distribution and, thus, the quantiles of assets' returns. The CoVaR measure by Adrian and Brunnermeier (2016) neglects such a phenomenon, as (1)—(2) do not include any factor reflecting the heteroskedastic behavior of the returns' densities and, thus, of both $Q^{(i)}_{\theta}(y_t)$ and $Q_{\tau}(x_{i,t})$. Here, we bridge this gap by adding autoregressive components capturing the quantiles' dynamics over time, inspired by the Conditional Autoregressive Value-at-Risk (CaViaR) introduced by Engle and Manganelli (2004). The heteroskedasticity in the CoVaR's dynamics is also considered, under a different approach, based on GARCH modelling, by Girardi and Ergun (2013).

We then rewrite (1)—(2) as follows:

$$Q_{\tau}(x_{i,t}) = \alpha^{(i)}_{\tau} + \phi^{(i)}_{1,\tau} Q_{\tau}(x_{i,t-1}) + \phi^{(i)}_{2,\tau} f(x_{i,t-1}) + \boldsymbol{\beta}^{(i)}_{\tau} \mathbf{M}'_{t-1}, \tag{6}$$

$$Q^{(i)}_{\theta}(y_t) = \delta^{(i)}_{\theta} + \psi^{(i)}_{1,\theta} Q^{(i)}_{\theta}(y_{t-1}) + \psi^{(i)}_{2,\theta} f(y_{t-1}) + \lambda^{(i)}_{\theta} x_{i,t} + \boldsymbol{\gamma}^{(i)}_{\theta} \mathbf{M}'_{t-1}. \tag{7}$$

The latent autoregressive components $Q_{\tau}(x_{i,t-1})$ and $Q^{(i)}_{\theta}(y_{t-1})$ smooth the changes in the estimated quantiles over time, capturing their heteroscedasticity. Moreover, $\phi^{(i)}_{2,\tau}$ and $\psi^{(i)}_{2,\theta}$ link $Q_{\tau}(x_{i,t})$ and

[1] See for instance Figure 6.

$Q_\theta^{(i)}(y_t)$ to the past of $x_{i,t}$ and y_t, respectively. Among all the possible functions, we set $f(x_{i,t-1}) = |x_{i,t-1}|$ and $f(y_{t-1}) = |y_{t-1}|$, as suggested by Engle and Manganelli (2004). Such a choice implies a direct response of the quantiles to the returns processes, symmetrically treating the effect of positive and negative returns on VaR. (6)—(7) correspond to CaViaR specifications augmented by control variables. In addition, (7) includes also the i-th company's return.

Engle and Manganelli (2004) discuss the estimation of the CaViaR model, a nonlinear quantile autoregression, proving that the estimators of the CaViaR's parameters are consistent and asymptotically normal. We follow Engle and Manganelli (2004) for estimating the parameters in (6)—(7) and their standard errors. Other works in the literature that study, under different viewpoints, quantile autoregressions are, for instance, Koenker and Xiao (2004) and Li et al. (2015).

We then introduce the Conditional CaViaR (CoCaViaR) as the generalization of the CoVaR accounting for the presence of autoregressive terms in (6)—(7). The CoCaViaR is defined as:

$$CoCaViaR_{t,\theta,\tau}^{(i)} = \widehat{\delta}_\theta^{(i)} + \widehat{\psi}_{1,\theta}^{(i)} \widehat{Q}_\theta^{(i)}(y_{t-1}) + \widehat{\psi}_{2,\theta}^{(i)} |y_{t-1}| + \widehat{\lambda}_\theta^{(i)} \widehat{Q}_\tau(x_{i,t}) + \widehat{\boldsymbol{\gamma}}_\theta^{(i)} \mathbf{M}'_{t-1}, \tag{8}$$

where $\widehat{Q}_\tau(x_{i,t}) = \widehat{\alpha}_\tau^{(i)} + \widehat{\phi}_{1,\tau}^{(i)} \widehat{Q}_\tau(x_{i,t-1}) + \widehat{\phi}_{2,\tau}^{(i)} |x_{i,t-1}| + \widehat{\boldsymbol{\beta}}_\tau^{(i)} \mathbf{M}'_{t-1}$ is estimated from (6).

The CoCaViaR of the financial system can be also conditioned to the median state of the i-th company by setting $\tau = 1/2$ and obtaining $CoCaViaR_{t,\theta,1/2}^{(i)}$. Again, we set $\theta = \tau$ in our empirical analysis and measure the marginal contribution of the conditioning company to the systemic risk through the ΔCoCaViaR:

$$\Delta CoCaViaR_{t,\tau}^{(i)} = CoCaViaR_{t,\theta,\tau}^{(i)} - CoCaViaR_{t,\theta,1/2}^{(i)} = \widehat{\lambda}_\theta^{(i)} \left[\widehat{Q}_\tau(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t}) \right]. \tag{9}$$

Despite taking the same form, that is, $\widehat{\lambda}_\theta^{(i)} \left[\widehat{Q}_\tau(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t}) \right]$, $\Delta CoVaR_{t,\tau}^{(i)}$ and $\Delta CoCaViaR_{t,\tau}^{(i)}$ are computed from different models, with potential differences due to the presence of the autoregressive components in (6)—(7). This leads to differences in the estimation of the parameter $\lambda_\theta^{(i)}$ as well as in the conditional quantiles of the financial institution. In fact, we estimate $\lambda_\theta^{(i)}$ from $Q_\theta^{(i)}(y_t) = \delta_\theta^{(i)} + \psi_{1,\theta}^{(i)} Q_\theta^{(i)}(y_{t-1}) + \psi_{2,\theta}^{(i)} |y_{t-1}| + \lambda_\theta^{(i)} x_{i,t} + \boldsymbol{\gamma}_\theta^{(i)} \mathbf{M}'_{t-1}$ for the CoCaViaR. In contrast, we recover $\lambda_\theta^{(i)}$ from $Q_\theta^{(i)}(y_t) = \delta_\theta^{(i)} + \lambda_\theta^{(i)} x_{i,t} + \boldsymbol{\gamma}_\theta^{(i)} \mathbf{M}'_{t-1}$ for the CoVaR. Therefore, it is interesting to assess whether and in what measure $\lambda_\theta^{(i)}$ changes after including the CaViaR components in the CoCaViaR. Besides, we estimate $Q_\tau(x_{i,t})$ from $Q_\tau(x_{i,t}) = \alpha_\tau^{(i)} + \phi_{1,\tau}^{(i)} Q_\tau(x_{i,t-1}) + \phi_{2,\tau}^{(i)} |x_{i,t-1}| + \boldsymbol{\beta}_\tau^{(i)} \mathbf{M}'_{t-1}$ in case of the CoCaViaR, and from $Q_\tau(x_{i,t}) = \alpha_\tau^{(i)} + \boldsymbol{\beta}_\tau^{(i)} \mathbf{M}'_{t-1}$ in case of the CoVaR. Then, we expect that the inclusion of the CaViaR components lead to differences between the two approaches. Similar considerations apply to $Q_{1/2}(x_{i,t})$.

2.3 Conditional Quantile-Located Value-at-Risk

The CoVaR proposed by Adrian and Brunnermeier (2016) measures the Value-at-Risk of the financial system conditional to the distress state of a given company. The link between the VaR of the system and the VaR of the company consists in plugging $\widehat{Q}_\tau(x_{i,t})$, estimated from (1), into (2), obtaining the CoVaR in (3). Nevertheless, the parameters in (2), and then the coefficients in (3), are functions of θ only—they link the covariates to the θ-th quantile of y_t—without considering the role of τ, that is, the reference quantile for the conditioning financial institution. In other words, the estimation process behind (3) depends on $x_{i,t}$ and not on $Q_\tau(x_{i,t})$. As a result, the observations in the support of $x_{i,t}$ are equally weighted.

However, measures of extreme risk quantify the losses occurring during tail events, such as financial crises. In such periods the correlations among financial institutions increase and the risk of contagion threatens the stability of the entire economy, due to potential spillover effects. Consequently, the information content of extreme risk measures becomes increasingly relevant when we accentuate the distress degree in the financial connections. In particular, in the context of the CoVaR proposed by Adrian and Brunnermeier (2016), we can increase the distress degree by linking the left tails of the distributions of y_t and $x_{i,t}$ in the estimation process. In other words, we can estimate the parameters in (2) assuming that the financial system and the i-th company simultaneously lie in the left tails of their distributions. Therefore, we restrict the attention on the impact exerted by $x_{i,t}$ on $\widehat{Q}_\theta^{(i)}(y_t)$ in the neighbourhood of its τ-th quantile. This allows for a further degree of flexibility, as the parameters monitoring the impact of $x_{i,t}$ on y_t might depend on the location of both $x_{i,t}$ and y_t along their marginal support.

We clarify our viewpoint with a simple example. Let us consider two financial companies: A and B. The first is a small company (a regional bank, or a financial service company), while the second is a large financial agglomerate company (a large retail bank or a large investment bank). We might easily postulate that A and B have different impact on the market. Moreover, we might even claim that the impact of A on the market is not influenced by the fact that A lies on its median values or on an extreme quantile. On the contrary, it would be difficult to assert that B, which most likely plays a relevant role in the economy, has the same effects on the market's returns (or on the distribution of the system's returns) regardless of the state (median or stressed) in which it is. In fact, we expect the financial distress of B to have more serious consequences on the entire market when compared to a similar situation for A. Therefore, we might state that the impact of B on the market also depends on the state in which B is located. Nevertheless, the CoVaR and the CoCaViaR do not capture such impact.

We introduce the Conditional Quantile-Located Value-at-Risk (QL-CoVaR) to overcome this limitation. In building such a risk measure, we follow Sim and Zhou (2015), which used a weighted quantile regression model, with kernel-based weights, to estimate the relations in quantiles between oil prices and stock returns. Such approach corresponds to the nonparametric quantile regression, where the knots used to obtain the local quantiles are fixed at specific quantiles of $x_{i,t}$ (Koenker, 2005). The model we propose is then defined as follows:

$$Q^{(i)}_{\theta,\tau}(y_t) = \delta^{(i)}_{\theta,\tau} + \lambda^{(i)}_{\theta,\tau} x_{i,t} + \boldsymbol{\gamma}^{(i)}_{\theta,\tau} \mathbf{M}'_{t-1}. \tag{10}$$

In contrast to the models described so far, the parameters in (10) have both θ and τ as subscripts, as they depend on the quantiles levels of both y_t and $x_{i,t}$. In fact, the unknown parameters in (10) are estimated from the following minimization problem:

$$\underset{\delta^{(i)}_{\theta,\tau}, \lambda^{(i)}_{\theta,\tau}, \boldsymbol{\gamma}^{(i)}_{\theta,\tau}}{\arg\min} \sum_{t=1}^{T} \rho_\theta \left[y_t - \delta^{(i)}_{\theta,\tau} - \lambda^{(i)}_{\theta,\tau} x_{i,t} - \boldsymbol{\gamma}^{(i)}_{\theta,\tau} \mathbf{M}'_{t-1} \right] K \left(\frac{\widehat{F}_{t|t-1}(x_{i,t}) - \tau}{h} \right), \tag{11}$$

where $\rho_\theta(e) = e(\theta - \mathbf{1}_{\{e<0\}})$ is the asymmetric loss function used in the quantile regression method by Koenker and Bassett (1978); $\mathbf{1}_{\{\cdot\}}$ is an indicator function, taking the value of 1 if the condition in $\{\cdot\}$ is satisfied, the value of 0 otherwise; $K(\cdot)$ is the kernel function, with bandwidth h, whereas $\widehat{F}_{t|t-1}(x_{i,t})$ is the empirical conditional quantile of $x_{i,t}$.

Let us then focus on the procedure we propose to estimate $\widehat{F}_{t|t-1}(x_{i,t})$. First of all, we estimate a large set of $x_{i,t}$'s quantiles in the support $\tau \in (0,1)$ from the quantile regression model (1). Note that the $Q_\tau(x_{i,t})$ values estimated for $\tau \in (0,1)$ are no longer interpreted as Values-at-Risk of $x_{i,t}$, given that we consider the entire distribution of $x_{i,t}$, not only the left tail. When estimating multiple quantiles in the interval $(0,1)$, the standard quantile regression approach by Koenker and Bassett (1978), that estimates individual quantiles, does not guarantee their coherence, i.e. their monotonicity for $\tau \in (0,1)$ (e.g. we might obtain $\widehat{Q}_{0.95}(x_{i,t}) < \widehat{Q}_{0.90}(x_{i,t})$). In order to obtain a valid conditional distribution of $x_{i,t}$, we use the method developed by Bondell et al. (2010) by which we obtain a large set of quantiles having a monotonic behavior for $\tau \in (0,1)$. Then, we linearly interpolate the set of quantiles to obtain the conditional distribution of $x_{i,t}$ at time t, denoted as $\widehat{F}(x_{i,t}|\mathbf{M}'_{t-1})$. Finally, we recover $\widehat{F}_{t|t-1}(x_{i,t})$, as the probability level, extrapolated from $\widehat{F}(x_{i,t}|\mathbf{M}'_{t-1})$, corresponding to the realization $x_{i,t}$. On the basis of a rolling window procedure, we estimate $\widehat{F}_{t|t-1}(x_{i,t})$ for each $t = ws, ws+1, ..., T$, where ws denotes the length of the estimation window. In general, a larger ws improves the statistical properties of the quantiles $\widehat{Q}_\tau(x_{i,t})$, for $\tau \in (0,1)$, and provide more stable estimates. Besides, a larger set of estimated quantiles $\widehat{Q}_\tau(x_{i,t})$ allows to obtain a more accurate conditional distribution $\widehat{F}(x_{i,t}|\mathbf{M}'_{t-1})$, reducing the errors coming from the interpolation. Nevertheless, increasing both the length of the estimation window and the number of quantiles leads to sensibly higher costs in terms of computational burden.

Here, in deriving $\widehat{F}_{t|t-1}(x_{i,t})$ we differ from Sim and Zhou (2015) who, instead, rely on the unconditional (full sample) empirical quantile:

$$\widehat{F}(x_{i,t}) = T^{-1} \sum_{k=1}^{T} \mathbf{1}_{\{x_{i,k} < x_{i,t}\}} \tag{12}$$

in place of $\widehat{F}_{t|t-1}(x_{i,t})$.

Therefore, the use of (12) implicitly relies on the stability of quantiles over time, neglecting the fact that financial returns are typically affected by heteroskedasticity and other elements impacting on the location, scale and symmetry of their distributions.[2] In contrast, $\widehat{F}_{t|t-1}(x_{i,t})$ builds on a dynamic conditional distribution of $x_{i,t}$ and then captures the heteroskedastic behavior and, in general, the instability of the $x_{i,t}$'s distribution over time.

The estimation process in (11) relies on the kernel bandwidth value h: the smaller h, the smaller is the bias of the estimates, but the larger is their variance, and vice versa. In contrast to Sim and Zhou (2015), which report results for a bandwidth value equal to 0.05, in the robustness analysis we test the sensitivity of the estimates for different h values. Then, we check whether, and in what measure, the results change according to the choice of the bandwidth value h. As for the computation of the standard errors of the coefficients in (10), we suggest again to use the bootstrap approach (Efron, 1979), implementing the xy-pair method (Kocherginsky, 2003).

From the method described above, we compute the QL-CoVaR at the τ-th level as follows:

$$QL\text{-}CoVaR^{(i)}_{t,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1}, \tag{13}$$

where $\widehat{Q}_{\tau}(x_{i,t}) = \widehat{\alpha}^{(i)}_{\tau} + \widehat{\boldsymbol{\beta}}^{(i)}_{\tau}\mathbf{M}'_{t-1}$.

Then, given $\theta = \tau$, and evaluating the model also for $\tau = 1/2$, we define the ΔQL-CoVaR as:

$$\begin{aligned}\Delta QL\text{-}CoVaR^{(i)}_{t,\tau} &= QL\text{-}CoVaR^{(i)}_{t,\theta,\tau} - QL\text{-}CoVaR^{(i)}_{t,\theta,1/2} = \widehat{\delta}^{(i)}_{\theta,\tau} - \widehat{\delta}^{(i)}_{\theta,1/2} + \widehat{\lambda}^{(i)}_{\theta,\tau}\left[\widehat{Q}_{\tau}(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t})\right] \\ &+ (\widehat{\lambda}^{(i)}_{\theta,\tau} - \widehat{\lambda}^{(i)}_{\theta,1/2})\widehat{Q}_{1/2}(x_{i,t}) + (\widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau} - \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2})\mathbf{M}'_{t-1}. \end{aligned}\tag{14}$$

As expected, $\Delta QL\text{-}CoVaR^{(i)}_{t,\tau}$ includes more components than $\Delta CoVaR^{(i)}_{t,\tau}$ in (5) and $\Delta CoCaViaR^{(i)}_{t,\tau}$ in (9). By exploiting the informative content of these additional components, we recover further information about the relations between the financial system and the single financial companies, in particular when the focus is placed on the left tails of their distributions.[3]

[2]This evidence is confirmed in our dataset by using the DQ and the SQ tests developed by Qu (2008). The DQ test checks for the presence of structural breaks in the conditional distribution of a given variable. In contrast, the SQ test verifies the presence of structural breaks at specific quantiles levels (in our analysis, we test the 5%, the 50% and the 95% levels). As expected, both the DQ and the SQ provide evidence against the assumption of stable quantiles and distributions over time, therefore we prefer not to use $F_T(x_{i,t})$ defined in (12) because it does not capture such dynamics. The dataset is described in Section 3, whereas the output of both the DQ and the SQ tests are available on request.

[3]The derivation of $\Delta QL\text{-}CoVaR^{(i)}_{t,\tau}$ in (14) is reported in Appendix A.

2.4 Conditional Autoregressive Quantile-Located Value-at-Risk

The combination between the CoCaViaR in (8) and the QL-CoVaR in (13) leads to the Conditional Autoregressive Quantile-Located Value-at-Risk (QL-CoCaViaR). Note that the Quantile-Located term emphasizes the focus on the impact exerted by $x_{i,t}$ on y_t when both variables are in a low quantile of their marginal distribution (see Section 2.3). The Autoregressive term refers to the heteroskedastic behavior of the quantiles over time (see Section 2.2).

The first step consists in estimating $Q^{(i)}_{\theta,\tau}(y_t)$ conditional to the distress state of $x_{i,t}$:

$$Q^{(i)}_{\theta,\tau}(y_t) = \delta^{(i)}_{\theta,\tau} + \psi^{(i)}_{1,\theta,\tau} Q^{(i)}_{\theta,\tau}(y_{t-1}) + \psi^{(i)}_{2,\theta,\tau}|y_{t-1}| + \lambda^{(i)}_{\theta,\tau} x_{i,t} + \boldsymbol{\gamma}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1}, \tag{15}$$

where the parameters are estimated from the following minimization problem:

$$\begin{aligned} \underset{\delta^{(i)}_{\theta,\tau},\psi^{(i)}_{1,\theta,\tau},\psi^{(i)}_{2,\theta,\tau},\lambda^{(i)}_{\theta,\tau},\boldsymbol{\gamma}^{(i)}_{\theta,\tau}}{\arg\min} \quad & \sum_{t=1}^{T} \rho_\theta \left[y_t - \delta^{(i)}_{\theta,\tau} - \psi^{(i)}_{1,\theta,\tau} Q^{(i)}_{\theta,\tau}(y_{t-1}) - \psi^{(i)}_{2,\theta,\tau}|y_{t-1}| - \lambda^{(i)}_{\theta,\tau} x_{i,t} - \boldsymbol{\gamma}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1} \right] \\ \times \quad & K\left(\frac{\widehat{F}_{t|t-1}(x_{i,t}) - \tau}{h} \right). \end{aligned} \tag{16}$$

We estimate the parameters in (15) and their standard errors by modifying the algorithm proposed by Engle and Manganelli (2004) for the CaViaR, to take into account the smoothing effect of the kernel function $K(\cdot)$. Then, after estimating the parameters in (15) and setting $\theta = \tau$, we compute the QL-CoCaViaR at the τ-th level as follows:

$$\textit{QL-CoCaViaR}^{(i)}_{t,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\psi}^{(i)}_{1,\theta,\tau} \widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) + \widehat{\psi}^{(i)}_{2,\theta,\tau}|y_{t-1}| + \widehat{\lambda}^{(i)}_{\theta,\tau} \widehat{Q}_\tau(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1}, \tag{17}$$

where $\widehat{Q}_\tau(x_{i,t}) = \widehat{\alpha}^{(i)}_\tau + \widehat{\phi}^{(i)}_{1,\tau} \widehat{Q}_\tau(x_{i,t-1}) + \widehat{\phi}^{(i)}_{2,\tau}|x_{i,t-1}| + \widehat{\boldsymbol{\beta}}^{(i)}_\tau \mathbf{M}'_{t-1}$.

The evaluation of the impact exerted by a financial company in distress on the system, as in the original CoVaR framework, becomes now quite complex due to the joint presence of the autoregressive and the quantile-located components. Then, Subsection 2.5 provides the directions for disentangling and interpreting the outcomes of the ΔQL-CoCaViaR.

2.5 Interpreting the ΔCoCaViaR estimates

The complex structure of the QL-CoCaViaR might challenge the financial interpretation of the model's estimates. We show how the model provides a rich and financially relevant interpretation of the link between a financial company in a stressed state and the financial system using a companion representation. We focus here only on the QL-CoCaViaR, the most general model, that nests all the proposed specifications and considers the CoVaR of Adrian and Brunnermeier (2016) as baseline.

Given $\theta = \tau$, the ΔQL-CoCaViaR can be rewritten as follows:[4]

$$
\begin{aligned}
\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)} &= QL\text{-}CoCaViaR_{t,\theta,\tau}^{(i)} - QL\text{-}CoCaViaR_{t,\theta,1/2}^{(i)} \\
&= \underbrace{\left(\widehat{\delta}_{\theta,\tau}^{(i)} - \widehat{\delta}_{\theta,1/2}^{(i)}\right)}_{c_{1,\tau}^{(i)}} + \underbrace{\widehat{\lambda}_{\theta,\tau}^{(i)} \left[\widehat{Q}_{\tau}(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t})\right]}_{c_{2,t,\tau}^{(i)}} \\
&+ \underbrace{(\widehat{\lambda}_{\theta,\tau}^{(i)} - \widehat{\lambda}_{\theta,1/2}^{(i)})\widehat{Q}_{1/2}(x_{i,t})}_{c_{3,t,\tau}^{(i)}} + \underbrace{(\widehat{\boldsymbol{\gamma}}_{\theta,\tau}^{(i)} - \widehat{\boldsymbol{\gamma}}_{\theta,1/2}^{(i)})\mathbf{M}_{t-1}'}_{c_{4,t,\tau}^{(i)}} \\
&+ \underbrace{(\widehat{\psi}_{2,\theta,\tau}^{(i)} - \widehat{\psi}_{2,\theta,1/2}^{(i)})|y_{t-1}|}_{c_{5,t,\tau}^{(i)}} + \underbrace{\left[\widehat{\psi}_{1,\theta,\tau}^{(i)} - \widehat{\psi}_{1,\theta,1/2}^{(i)}\right]\widehat{Q}_{\theta,\tau}^{(i)}(y_{t-1})}_{c_{6,t,\tau}^{(i)}} \\
&+ \underbrace{\widehat{\psi}_{1,\theta,1/2}^{(i)} \left[\widehat{Q}_{\theta,\tau}^{(i)}(y_{t-1}) - \widehat{Q}_{\theta,1/2}^{(i)}(y_{t-1})\right]}_{c_{7,t,\tau}^{(i)}}.
\end{aligned}
\tag{18}
$$

The decomposition in (18) allows to shed some light on the financial interpretation of the different ΔQL-CoCaViaR's components:

- $c_{1,\tau}^{(i)}$ measures the shift in location of the system's density along the distribution of the i-th company, for $i = 1, ..., N$; it quantifies the change in the system risk, when the financial company moves from the median state to the τ-th quantile. Such change is not attributable to the covariates or to the change in the financial company risk measures;
- $c_{2,t,\tau}^{(i)}$ captures the effect of the increment in the i-th company's riskiness; this corresponds to the standard ΔCoVaR estimated under the distress state of both the system and the i-th company;
- $c_{3,t,\tau}^{(i)}$ quantifies the contribution due to the change in the impact of the i-th company on the financial system. It provides a measure of the change of the system's sensitivity to the financial company. Assuming a positive conditional median for a given financial institution, positive values of $c_{3,t,\tau}^{(i)}$ suggest a larger sensitivity of the system to the i-th company's returns when those are in their lower quantiles;
- $c_{4,t,\tau}^{(i)}$ is the contribution associated to changes in the impact of the control variables to the system; it quantifies the effect of changes in the sensitivity of the system to the covariates when the financial company's returns move from the median to the lower quantiles. Non-null values would signal changes in the system reaction to the control variables;
- $c_{5,t,\tau}^{(i)}$ measures the change in the relevance of the past system's absolute returns. Non-null values of this component highlight a change in the impact of the past system's returns on the θ-th system's quantile, when we compare the system quantile conditioning on the financial company return being in a median or in a lower quantile;

[4] The derivation of the ΔQL-CoCaViaR is reported in Appendix B.

- $c^{(i)}_{6,t,\tau}$ allows to assess the change in the persistence of the system's quantiles. Non-null values of $c^{(i)}_{6,t,\tau}$ suggest that the system changes its dependence on its past quantiles when contrasting the system's quantiles conditional to a median or a stressed state of the i-th company;
- $c^{(i)}_{7,t,\tau}$ measures the impact of the lagged change in the risk of the system computed conditional to the i-th company being in a median or in a stressed state. Non-null values of $c^{(i)}_{7,t,\tau}$ suggest that the system's past quantiles were different when conditioning on different quantiles of the financial company's returns.

After disentangling the seven components discussed above, it is interesting to asses how and in what measure they contribute to the ΔQL-CoCaViaR. For instance, we might highlight relevant effects coming from the conditioning on both the system and a financial company quantile. The conditioning financial company impacts on the ΔQL-CoCaViaR under different components and, therefore, it will be important to check whether, for instance, $c^{(i)}_{2,t,\tau}$ or $c^{(i)}_{3,t,\tau}$ prevail over the other components. Similar considerations hold for the effects coming from the lagged quantiles of y_t, then it is important to compare $c^{(i)}_{6,t,\tau}$ and $c^{(i)}_{7,t,\tau}$.

By jointly focusing on the different $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$ determinants, we can isolate the impact of the elements we add to the approach proposed by Adrian and Brunnermeier (2016). In particular, if $c^{(i)}_{5,t,\tau} = c^{(i)}_{6,t,\tau} = 0$, there is no impact coming from the changes in the autoregressive components, hence the CaViaR parameters do not depend on the quantiles of the company. If, in addition, $\psi^{(i)}_{1,\theta,1/2} = 0$, the ΔQL-CoCaViaR loses its autoregressive behavior. If $c^{(i)}_{4,t,\tau} = 0$, the role of the control variables does not depend on the state of the company (or it is not influenced by the joint occurrence of the stress states in the company and in the system). If $c^{(i)}_{1,t,\tau} = c^{(i)}_{3,t,\tau} = 0$, the impact of the company does not depend on the location of the company return over its support, then we do not need to account for the quantile dependence in the evaluation of the ΔQL-CoCaViaR. Finally, if all the determinants in (18), $c^{(i)}_{2,t,\tau}$ excluded, are equal to zero, we are back to the original proposal of Adrian and Brunnermeier (2016), adjusted for the quantile-located effects, as $\lambda^{(i)}_{\theta,\tau}$ is now estimated under the joint distress state of the system and of the i-th company.

To test for the relevance of the CaViaR-like components and of the quantile-location effects, we can evaluate the contribution of the various ΔQL-CoCaViaR's determinants we highlighted above, comparing their size over time and/or in the cross-sectional dimension. Notably, this latter element is also useful when the parameters' estimates are statistically significant but we do want to monitor the economic relevance of each ΔQL-CoCaViaR's component.

2.6 Backtesting procedures

We evaluate the out-of-sample performance of the risk measures described in Sections 2.1—2.4, namely the CoVaR, the CoCaViaR, the QL-CoVaR and the QL-CoCaViaR, by means of selected statistical tests and loss functions. To implement the out-of-sample analyses, we resort, as a standard practice, to a rolling window procedure. Given a window size of ws days ending at time t, we first estimate the parameters of the various models; see Equations (1), (2), (6), (7), (10) and (15). By using these estimates, conditional on the information set available at day t, we compute the out-of-sample risk measures as follows:

$$CoVaR^{(i)}_{t+1,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta} + \widehat{\lambda}^{(i)}_{\theta}\widehat{Q}_{\tau}(x_{i,t+1}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta}\mathbf{M}'_{t}, \tag{19}$$

$$CoCaViaR^{(i)}_{t+1,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta} + \widehat{\psi}^{(i)}_{1,\theta}\widehat{Q}^{(i)}_{\theta}(y_t) + \widehat{\psi}^{(i)}_{2,\theta}|y_t| + \widehat{\lambda}^{(i)}_{\theta}\widehat{Q}_{\tau}(x_{i,t+1}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta}\mathbf{M}'_{t}, \tag{20}$$

$$QL\text{-}CoVaR^{(i)}_{t+1,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t+1}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t}, \tag{21}$$

$$QL\text{-}CoCaViaR^{(i)}_{t+1,\theta,\tau} = \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\psi}^{(i)}_{1,\theta,\tau}\widehat{Q}^{(i)}_{\theta,\tau}(y_t) + \widehat{\psi}^{(i)}_{2,\theta,\tau}|y_t| + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t+1}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t}. \tag{22}$$

We remind the reader that we estimate $\widehat{Q}_{\tau}(x_{i,t+1})$ in (19) and in (21) from the quantile regression model:

$$\widehat{Q}_{\tau}(x_{i,t+1}) = \widehat{\alpha}^{(i)}_{\tau} + \widehat{\boldsymbol{\beta}}^{(i)}_{\tau}\mathbf{M}'_{t}. \tag{23}$$

In contrast, we estimate $\widehat{Q}_{\tau}(x_{i,t+1})$ in (20) and in (22) as follows:

$$\widehat{Q}_{\tau}(x_{i,t+1}) = \widehat{\alpha}^{(i)}_{\tau} + \widehat{\phi}^{(i)}_{1,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\phi}^{(i)}_{2,\tau}|x_{i,t}| + \widehat{\boldsymbol{\beta}}^{(i)}_{\tau}\mathbf{M}'_{t}. \tag{24}$$

We repeat this estimation procedure by rolling the estimation window with step of one day ahead, until we use all the available data. By comparing the quantile forecasts with the out-of-sample realizations of the market index y_t, for $t = ws + 1, ..., T$, we compute the following hit functions:

$$Hit^{\mathcal{M}|i}_{t,\theta,\tau} = \begin{cases} 1 & \text{if } y_t < \mathcal{M}^{(i)}_{t,\theta,\tau} \\ 0 & \text{otherwise} \end{cases}, \tag{25}$$

where $\mathcal{M}$ = {CoVaR, CoCaViaR, QL-CoVaR, QL-CoCaViaR}, that is, we have four different hit functions, one for each risk model; building on the hit functions (25), we first compare the accuracy of the competitive risk measures by implementing the Kupiec (1995) and the Christoffersen (1998) tests.

The measures (19)—(22) are conditional to the distress state of the individual companies, but the classical Kupiec (1995) and Christoffersen (1998) tests do not capture such a conditioning. In fact, they focus only on the quantile violation of the market index. Therefore, we adopt the method proposed by Girardi and Ergun (2013), that includes the distress state of the conditioning companies into the Kupiec (1995) and Christoffersen (1998) tests evaluated at the market index level. First, we consider the hit values of the i-th financial company, defined as:

$$Hit^{i}_{t,\tau} = \begin{cases} 1 & \text{if } x_{i,t} < \widehat{Q}_{\tau}(x_{i,t}) \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

In a second step, we compute the hit values of the system as in (25), and implement the Kupiec (1995) and Christoffersen (1998) tests for the days in which $Hit_{t,\tau}^i = 1$, that is, when the i-th company is in a stressed state. We refer the reader to Girardi and Ergun (2013) for further details on the tests implementation under a double conditioning (i.e. market index quantile violation and company under distress).

Note that we compute $\widehat{Q}_\tau(x_{i,t})$ from (23) when we consider the CoVaR and the QL-CoVaR models to quantify the risk associated to y_t. In contrast, we use (24) when we focus on the CoCaViaR and the QL-CoCaViaR models. As a result, we might obtain different sequences in (26) according to the risk measure we are using to monitor the system's risk. Therefore, in addition to the method described above, we also implement the test proposed by Girardi and Ergun (2013) on the basis of the following model-free hit function for the company distress:

$$Hit_{t,\tau}^i = \begin{cases} 1 & \text{if } x_{i,t} < \widehat{q}_\tau(x_i) \\ 0 & \text{otherwise} \end{cases}, \tag{27}$$

where $\widehat{q}_\tau(x_i)$ is the sample τ-th quantile of the company computed from the entire time series, that is, using the observations recorded in $t = 1, ..., T$. In this way, we evaluate the accuracy of the four risk measures—CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR—on the basis of the same series of company hits $Hit_{t,\tau}^i$.

In Girardi and Ergun (2013), the financial institution can be at most at its VaR to be in a stressed state, and this is coherent with their definition of CoVaR. In our case, we are coherent with the original definition of Adrian and Brunnermeier (2016), but we are also aware that the distress state of the company might not be confined to the case of observing company returns below a given quantile. In fact, following the previous hit functions, we completely disregard days where $x_{i,t}$ is greater than $\widehat{Q}_\tau(x_{i,t})$ (or $\widehat{q}_\tau(x_i)$) although $x_{i,t}$ lies in the right-neighbourhood of its τ-th quantile.

To increase the flexibility of the company distress state identification, and at the same time to remain within our modeling framework, we account for the impact exerted by $x_{i,t}$ in the neighbourhood of its τ-th quantile by replacing (26)—(27) with the following hit function:

$$Hit_{t,\theta}^i = \begin{cases} 1 & \text{if } K\left(\frac{\widehat{F}_{t|t-1}(x_{i,t})-\tau}{h}\right) > \nu \text{ or } x_{i,t} < \widehat{Q}_\tau(x_{i,t}) \\ 0 & \text{otherwise} \end{cases}, \tag{28}$$

where $K\left(\frac{\widehat{F}_{t|t-1}(x_{i,t})-\tau}{h}\right)$ is the same kernel function we use to estimate the parameters entering the quantile-located measures (see Sections 2.3—2.4), whereas ν is a given threshold. As a result, we do not lose the information about $x_{i,t}$ being close to its τ-th quantile from the right.[5]

[5]Note that the condition $Hit_{t,\theta}^i = 1$ if $K\left(\frac{\widehat{F}_{t|t-1}(x_{i,t})-\tau}{h}\right) > \nu$ *or* $x_{i,t} < \widehat{Q}_\tau(x_{i,t})$ is more pessimistic than $Hit_{t,\theta}^i = 1$

Finally, we evaluate the out-of-sample performance of the four risk measures by using a loss function. A typical choice in the literature (see, e.g. Caporin (2008)) builds on the square of the difference between the realization of the system and the quantile forecast. We slightly modify this approach to take into account the distress state of the conditioning company. In particular, we first define the following weighting function:

$$W_{t,\tau}^{i} = \begin{cases} 1 & \text{if } x_{i,t} \le \widehat{Q}_{\tau}(x_{i,t}) \\ e^{-u_{t,\tau}^{2}} & \text{otherwise} \end{cases}, \tag{29}$$

where $u_{t,\tau} = (x_{i,t} - \widehat{Q}_{\tau}(x_{i,t}))/h$.

Note that (29) is a continuous asymmetric function at $x_{i,t} = \widehat{Q}_{\tau}(x_{i,t})$ taking the value of 1 in the worst scenario, that is, when $x_{i,t} \le \widehat{Q}_{\tau}(x_{i,t})$, and smoothly decreasing for $x_{i,t} > \widehat{Q}_{\tau}(x_{i,t})$.

We thus introduce the loss function:

$$L_{t,\theta,\tau}^{\mathcal{M}|i} = \begin{cases} \left[1 + (y_t - \mathcal{M}_{t,\theta,\tau}^{(i)})^2\right] W_{t,\tau}^{i} & \text{if } y_t < \mathcal{M}_{t,\theta,\tau}^{(i)} \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

where $\mathcal{M} = \{$CoVaR, CoCaViaR, QL-CoVaR, QL-CoCaViaR$\}$. Using the loss functions across the four risk models, we can easily test for the equality of expected loss by resorting to a Diebold and Mariano (2002)-type test.

3 The data

Our dataset includes the daily returns of 1,155 U.S. financial institutions (952 banks and 203 insurance companies) in the period between October 10, 2000 and July 31, 2015, for a total of 3,864 days.[6] Not all the companies' time series span the entire period. In fact, some of them enter the dataset after October 10, 2000, whereas others exit before July 31, 2015. Henceforth, we consider as present all the companies for which we have the data at a given day between October 10, 2000 and July 31, 2015. Panel (a) of Figure 1 displays the number of banks and insurance companies present from October 10, 2000 until July 31, 2015, showing that the number of insurances is quite constant over time (with an average number equal to 88), whereas the number of banks increases as time passes, from 253 at October 10, 2000 to 543 at July 31, 2015. Moreover, we computed the difference between the number of the companies present at the end of a given month and the number of the companies present at the end of the previous month for the entire period. As shown by the histograms in Panel (b) of Figure 1, the distribution of these differences has a larger dispersion for the banks (interquartile range equal

if $K\left(\frac{\widehat{F}_{t|t-1}(x_{i,t})-\tau}{h}\right) > \nu$. Nevertheless, we checked that the results coming from the two hit functions are similar and are available upon request.

[6] The data are recovered from Thomson Reuters Datastream.

to 3) than for the insurances (interquartile range equal to 1) and the distribution of the banks has a remarkable right tail (skewness index equal to 0.62). The median of the differences is equal to 1 for the banks, equal to 0 for the insurances. Then, the banks are characterized by larger monthly increments than the insurances.

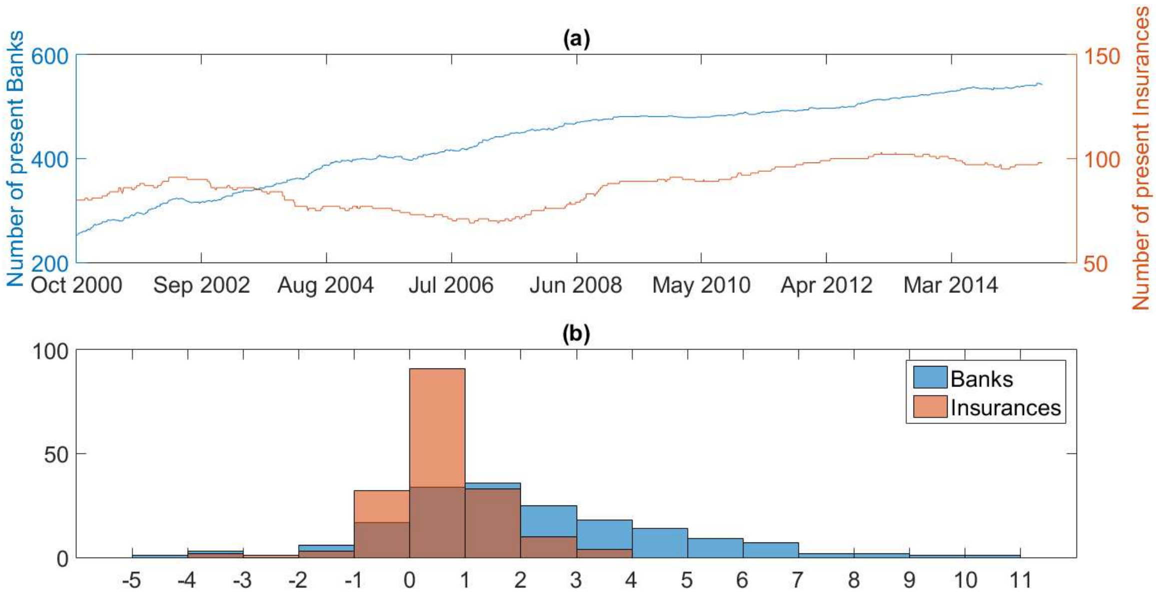

Figure 1: Panel (a) displays the number of banks and insurance companies present in our dataset for each day between October 10, 2000 and July 31, 2015. Panel (b) displays the histograms of the differences between the number of the companies (distinguished between banks or insurances) present at the end of a given month and the number of the companies present at the end of the previous month, for the entire period between October 10, 2000 and July 31, 2015.

To provide a more accurate description, we divided our dataset into 3 sub-periods, that is, October 10, 2000—September 15, 2005 (the first), September 16, 2005—August 25, 2010 (the second) and August 26, 2010—July 31, 2015 (the third). Descriptive statistics in Table 1 are computed for banks and insurances separately. For each company for which we have at least 50 observations, we computed the following statistics: 5-th and 95-th percentiles, median and interquartile range. The cross-sectional medians of these statistics are then reported in Table 1.

Table 1: Descriptive statistics of daily financial returns

	BANKS				INSURANCES			
PERIOD	5P	MED	95P	IQR	5P	MED	95P	IQR
10/10/2000 - 15/09/2005	-2.666	0.080	2.991	1.217	-2.773	0.050	2.982	1.651
16/09/2005 - 25/08/2010	-4.285	-0.028	4.157	1.640	-3.533	0.005	3.490	1.829
26/08/2010 - 31/07/2015	-2.859	0.058	3.034	1.622	-2.309	0.064	2.401	1.443

We divided the dataset into 3 sub-periods: October 10, 2000-September 15, 2005 (first), September 16, 2005-August 25, 2010 (second), August 26, 2010-July 31, 2015 (third). For each institution for which we have at least 50 observations, we computed the following descriptive statistics: 5-th (5P, %) and 95-th (95P, %) percentiles, median (MED, %) and interquartile range (IQR, %). Table 1 reports the cross-sectional medians of these statistics.

Between September 16, 2005 and August 25, 2010, the 5-th percentile and the median of the returns are lower than the ones in the other periods, while having the highest values of the 95-th percentile and of the interquartile range. This is due to the subprime crisis, highlighting the worsening of the companies' performance in terms of profitability and risk.

The models described in Section 2 are estimated for each of the financial companies present in our dataset for at least 200 trading days. Hence, due to the data availability, 1,030 out of 1,155 companies are used as conditioning companies $x_{i,t}$. We also build an index reproducing the behaviour of the financial system (y_t) from the returns of the 1,155 financial institutions, weighted by their market values, from October 10, 2000 to July 31, 2015. We compare y_t with the Standard & Poor's 500 index (S&P 500). Although y_t and S&P 500 follow a similar trend, the former has larger spikes, especially during the subprime crisis (see Figure 6 in Appendix D).

$\mathbf{M}_t$ includes control variables related to the bond, equity and real estate markets. They are listed as follows: i) the CBOE Volatility Index (VIX); ii) the liquidity spread (LS), computed as the difference between the three-month collateral repo rate and the three-month bill rate; iii) the change in the three-month Treasury bill rate (TB); iv) the change in the slope of the yield curve (YC), computed as the spread between the ten-year Treasury rate and the three-month bill rate; v) the change in the credit spread between BAA-rated bonds and the Treasury rate (CS), both with the ten years maturity; vi) the daily equity market return (EM); vii) the excess return of the real estate sector over the market return (RE).[7]

The first principal component (fpc_t) of the control variables in $\mathbf{M}_t$ explains 96.50% of the variability in the data. fpc_t is driven by the value of VIX, as their correlation coefficient is equal to 0.99. fpc_t and VIX_t have different locations, as their medians are equal to -2.38 and 18.23, respectively; nevertheless, fpc_t and VIX_t have the same behavior in terms of higher-order moments.[8] We choose to use fpc_t in place of $\mathbf{M}_t$ for all models as it allows to exploit the almost totality of the information contained in $\mathbf{M}_t$ (fpc_t captures 96.50% of the variability, with still a focus on VIX). Reducing the dimensionality of $\mathbf{M}_t$ through fpc_t achieves relevant benefits in terms of computational burden and estimates' stability, especially when using both the CoCaViaR and the QL-CoCaViaR, which include latent autoregressive components.

4 Empirical findings

4.1 Empirical set-up and estimation

We estimate the four risk measures, that is, CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR, using two quantile levels—$\theta = \tau = 0.01$ and $\theta = \tau = 0.05$. As for the estimation of the QL-

[7]The control variables listed in i)—v) are taken from Thomson Reuters Datastream, whereas EM and RE are recovered from the industry portfolios built by Kenneth R. French, available at *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_ library.html*.

[8]See Figure 7 in Appendix D.

CoVaR's and the QL-CoCaViaR's parameters, we use the Gaussian kernel as $F(\cdot)$ by which we weight the observations of $x_{i,t}$ in (11) and in (16). Moreover, we run a sensitivity analysis and obtain the estimates at $h = \{0.10, 0.15, 0.20\}$, to assess whether, and in what measure, the results change according to the choice of h. As for $\widehat{F}_{t|t-1}(x_{i,t})$, given the values of $x_{i,t}$ and fpc_{t-1} recorded in $t = 2, ..., 51$, we estimate the quantile regression model in (1) for 50 quantile levels equally distributed—$\tau = \{0.01, 0.03, 0.05, ..., 0.99\}$—using the method proposed by Bondell et al. (2010). Therefore, we obtain 50 estimated quantiles $\widehat{Q}_\tau(x_{i,51}) = \widehat{\alpha}_\tau^{(i)} + \widehat{\beta}_\tau^{(i)} fpc_{50}$ having a monotonic behaviour in $\tau = \{0.01, 0.03, ..., 0.99\}$, that we interpolate to build the conditional distribution of $x_{i,51}$, denoted as $\widehat{F}(x_{i,51}|fpc_{50})$. We then obtain $\widehat{F}_{51|50}(x_{i,51})$—the probability level corresponding to the realization $x_{i,t}$ that we extrapolate from $\widehat{F}(x_{i,51}|fpc_{50})$. We use a rolling window procedure and estimate $\widehat{F}_{t|t-1}(x_{i,t})$ by updating for each $t = 52, ..., T$ the estimation window with the latest 50 values of $x_{i,t}$ and fpc_{t-1}.

We highlight that the conditional distribution $\widehat{F}(x_{i,t}|fpc_{t-1})$ is built by interpolating 50 conditional quantiles, the parameters of which are estimated from a sample of 50 observations. In general, increasing the sample size improves the asymptotic properties of the $Q_\tau(x_{i,t})$'s estimator. Furthermore, using a larger set of quantile levels in $\tau \in (0,1)$ improves the accuracy of the interpolation and, as a result, leads to more accurate estimates of $\widehat{F}(x_{i,t}|fpc_{t-1})$. Nevertheless, increasing both the sample size and the number of quantile levels is computationally expensive. After comparing for a subset of companies the results obtained with an estimation window of 50 observations and 50 interpolated quantiles with those obtained from a window of 200 observations and a grid of 99 quantiles—$\tau = \{0.01, 0.02, 0.03, ..., 0.99\}$. We choose the set-up with a smaller computational burden as the differences between the two approaches are negligible.

On the basis of the empirical set-up described above, we estimate the risk measures' parameters. For the sake of brevity, we summarize here the main findings, while further details are reported in Appendix C. As expected, in general, positive returns of the individual companies have a positive impact on the VaR of the financial system, as $\lambda_\theta^{(i)}$ (for CoVaR and CoCaViaR) and $\lambda_{\theta,\tau}^{(i)}$ (for QL-CoVaR and QL-CoCaViaR) take, on average, positive values. Notably, the relationships between the system and the individual companies become stronger when linking the left tails of their returns' distributions, that is, when focusing on the quantile-located effects. In fact, on average, $\lambda_{\theta,\tau}^{(i)}$ is greater than $\lambda_\theta^{(i)}$, highlighting that the system is more sensitive to the individual companies when accentuating the distress degree in their connections. On the other hand, including the CaViaR's components absorb, in part, the sensitivity of the system to the individual companies. In fact, on average, $\lambda_{\theta,\tau}^{(i)}$ and $\lambda_\theta^{(i)}$ decrease when including the CaViaR's components. Focusing on the CaViaR's components, the lagged quantile of the system—$Q_\theta^{(i)}(y_{t-1})$ for CoCaViaR and $Q_{\theta,\tau}^{(i)}(y_{t-1})$ for QL-CoCaViaR—has a positive impact on its current value ($Q_\theta^{(i)}(y_t)$ or $Q_{\theta,\tau}^{(i)}(y_t)$). This is an expected result due to the persistence of quantiles over time (Engle and Manganelli, 2004). In contrast, the second CaViaR's component, that is, $|y_{t-1}|$ has a negative impact on the lower quantiles of the system. Therefore, a very bad or a very good day symmetrically increase the probability of observing greater losses in the next day, consistent with the hypothesis laid down by Engle and Manganelli (2004). Finally, the control variable fpc_t has a negative impact on the system's quantiles. This is again an expected result, as fpc_t is driven by the VIX index (see Section 3). Therefore, the greater the VIX level the higher the market's volatility or risk, with negative effects on the y_t's lower quantiles.

4.2 A comparative analysis of competing systemic risk measures

We compute for each day and for each company of our dataset the ΔCoVaR and the new measures: ΔCoCaViaR, ΔQL-CoVaR and ΔQL-CoCaViaR. We first display in Figure 2 the trend of the cross-sectional median of each risk measure, distinguishing banks from insurances.[9] The ΔCoCaViaR is, on average, more conservative than the ΔCoVaR. Likewise, the ΔQL-CoCaViaR is more conservative than the ΔQL-CoVaR. This is an expected result as the CaViaR's components in both the ΔCoCaViaR and the ΔQL-CoCaViaR absorb, in part, the relations between the system and the financial companies (see Sections 4.1 and C). The quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—point out a greater extreme risk than ΔCoVaR and ΔCoCaViaR. This is again an expected result as ΔQL-CoVaR and ΔQL-CoCaViaR are estimated under the joint stressed state of the system and of the individual companies. Notably, the distance between the quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—and the other measures—ΔCoVaR and ΔCoCaViaR—is relatively low during tranquil periods and becomes accentuated during stressed phases—see, e.g., the period of the sub-prime crisis.

Smaller but still relevant spikes are observed during the 'internet bubble' in 2000, after the terrorist attacks in September 2001, around the stock market crash in 2002, during the war in Iraq (years 2001—2003) and during the European sovereign debt crisis (years 2010—2011). We then highlight the capability of the quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—to react more strongly during events of system-wide relevance.

From the systemic risk perspective, the values of the quantile-located risk measures are, in absolute terms, greater than the standard CoVaR. Nevertheless, these differences do not correspond to simple shifts. In Figure 3 we display the cross-sectional medians of the ratios ΔCoCaViaR/ΔCoVaR, ΔQL-CoVaR/ΔCoVaR and ΔQL-CoCaViaR/ΔCoVaR. We observe that the introduction of the CaViaR's components has a limited effect. In fact, the ratio ΔCoCaViaR/ΔCoVaR is substantially stable over time and close to one. In contrast, moving towards measures accounting for the stressed state of both the individual companies and the financial system leads to two different effects. First, we observe an increase in the systemic risk as we have already pointed out: the ratios ΔQL-CoVaR/ΔCoVaR and ΔQL-CoCaViaR/ΔCoVaR are always above one. Second, the ratios are not stable over time and tend to increase during period of market stress. Therefore, the introduction of quantile-location effects

[9] The systemic risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$. The plots obtained from other values of $\theta = \tau$ and h are qualitatively very similar and are available upon request.

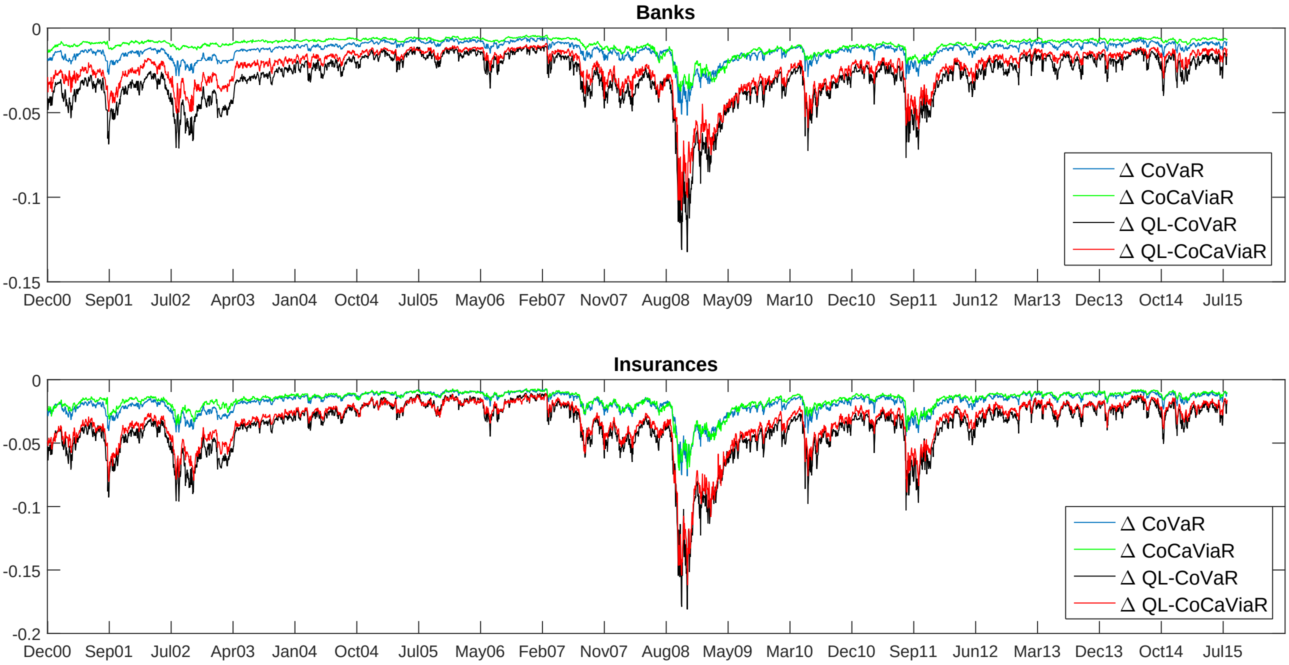

Figure 2: For each day in the period December 2000—July 2015, the figure displays the trend of the cross-sectional medians of the following risk measures: ΔCoVaR, ΔCoCaViaR, ΔQL-CoVaR and ΔQL-CoCaViaR, distinguishing banks from insurances. The risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$. The results obtained at $\theta = \tau = 0.05$ and $h = \{0.10, 0.20\}$ are qualitative similar and available upon request.

does not simply provide a shift in the measured systemic impact but also accounts for the change in the structural relation between the financial companies and the market when the overall system is in turmoil.

For each company we computed the 5-th and the 95-th percentiles, the median and the interquartile range of each risk measure's time series. We then calculated the cross-sectional medians of these statistics, as reported in Table 2. Including the CaViaR's components brings some differences between the ΔCoVaR and the ΔCoCaViaR. Nevertheless, the sign of these differences is not constant and depends on the θ levels. For instance, at $\theta = \tau = 0.01$, $\Delta CoVaR_{t,\tau}^{(i)}$ is greater, in absolute value, than $\Delta CoCaViaR_{t,\tau}^{(i)}$ in its 5-th and 95-th percentiles and at the median level. Hence, at $\theta = \tau = 0.01$, $\Delta CoVaR_{t,\tau}^{(i)}$ points out a greater marginal contribution of the companies to the systemic risk with respect to $\Delta CoCaViaR_{t,\tau}^{(i)}$. The opposite phenomenon holds at $\theta = \tau = 0.05$: $\Delta CoCaViaR_{t,\tau}^{(i)}$ reflects a greater risk with respect to $\Delta CoVaR_{t,\tau}^{(i)}$. Furthermore, $\Delta CoCaViaR_{t,\tau}^{(i)}$ is slightly more volatile than $\Delta CoVaR_{t,\tau}^{(i)}$, as we can see from the interquartile range.

All the statistics in Table 2 sensibly increase, in absolute value, when considering ΔQL-CoVaR and ΔQL-CoCaViaR, which reflect a larger systemic risk contribution coming from the financial companies. This is not surprising as $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ and $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$ are estimated under the distress state of both the system and the companies. Besides, on average, $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ is greater (in absolute value) than $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$, as the inclusion of the CaViaR's components in $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$

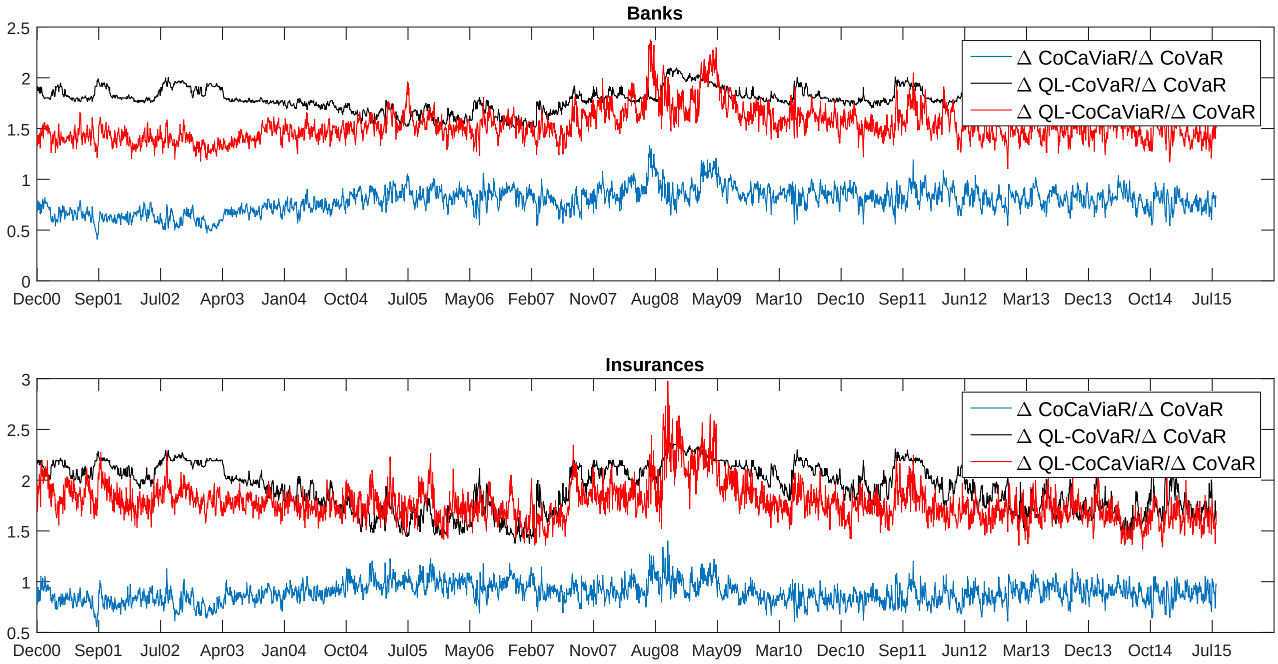

Figure 3: For each day in the period December 2000—July 2015, the figure displays the cross-sectional medians of the ratios ΔCoCaViaR/ΔCoVaR, ΔQL-CoVaR/ΔCoVaR and ΔQL-CoCaViaR/ΔCoVaR, distinguishing banks from insurances. The risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$. The results obtained at $\theta = \tau = 0.05$ and $h = \{0.10, 0.20\}$ are qualitative similar and available upon request.

absorbs, in part, the relationships between the system and the companies, lowering the contribution of the firms to the systemic risk, consistent with the analysis of the risk measures' coefficients in Section 4 and Appendix C. In contrast with the other measures, the estimates of $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ and $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$ are located, day by day, within the neighbourhood of the τ-th quantile of $x_{i,t}$. As discussed in Section 2.3, the quantiles (and thus the distribution) of $x_{i,t}$ change over time and these variations contribute to the volatility of both $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ and $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$. As a result, the $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$'s and the $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$'s interquartile ranges are greater than the ones observed for $\Delta CoVaR_{t,\tau}^{(i)}$ and $\Delta CoCaViaR_{t,\tau}^{(i)}$.

Table 2: Statistics of ΔCoVaR, ΔCoCaViaR, ΔQL-COVaR and ΔQL-CoCaViaR

	$\theta = \tau = 0.01$				$\theta = \tau = 0.05$			
	5P	MED	95P	IQR	5P	MED	95P	IQR
$\Delta CoVaR_{t,\tau}$	-1.022	-0.645	-0.468	0.215	-0.496	-0.306	-0.228	0.095
$\Delta CoCaViaR_{t,\tau}$	-0.990	-0.488	-0.263	0.255	-0.839	-0.417	-0.229	0.218
$\Delta QL\text{-}CoVaR_{t,\tau}$	-2.815	-1.442	-0.741	0.970	-1.605	-0.704	-0.348	0.538
$\Delta QL\text{-}CoCaViaR_{t,\tau}$	-3.112	-1.088	0.003	1.251	-2.015	-0.613	-0.019	0.712

For each day and for each company of our dataset, we compute the four risk measures given in the first column at $\theta = \tau = \{0.01, 0.05\}$, setting $h = 0.15$ (the results obtained for other bandwidth values are similar and are available upon request). For each company, we then compute the 5-th percentile (5P, %), the median (MED, %), the 95-th percentile (95P, %) and the interquartile range (IQR, %) of the four risk measures' time series. Finally, we report from left to right the cross-sectional medians of these statistics.

As next step, we also compare the information content of the four systemic risk measures using the following indicator:

$$Z_{j,k,\tau} = \frac{1}{TN} \sum_{i=1}^{N} \sum_{t=1}^{T} \mathbf{1}_{\left\{\Delta Risk_{j,t,\tau}^{(i)} - \Delta Risk_{k,t,\tau}^{(i)} < -\phi\right\}}, \tag{31}$$

where $\Delta Risk_{j,t,\tau}^{(i)}$ and $\Delta Risk_{k,t,\tau}^{(i)}$ are the j-th and the k-th measures in the set $S_{t,\tau}^{(i)} = \{100 \cdot \Delta CoVaR_{t,\tau}^{(i)}, 100 \cdot \Delta CoCaViaR_{t,\tau}^{(i)}, 100 \cdot \Delta QL\text{-}CoVaR_{t,\tau}^{(i)}, 100 \cdot \Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}\}$, for $j = 1, ..., 4$ and $k = 1, ..., 4$, $\mathbf{1}_{\{\cdot\}}$ is an indicator function taking the value of 1 if the condition in $\{\cdot\}$ is true, the value of 0 otherwise, whereas ϕ is a given threshold. Obviously, $Z_{j,k,\tau} = 0$ if $k = j$.

$Z_{j,k,\tau}$ in (31) quantifies the proportion of times—over T and N—in which the j-th systemic risk measure signals a worsening in the riskiness with respect to the k-th indicator. We neglect minimal differences, therefore $Z_{j,k,\tau}$ captures a given worsening if and only if its magnitude is greater, in absolute value, than ϕ. Here, we set ϕ equal to 10 basis points and report the results—obtained at $\theta = \tau = \{0.01, 0.05\}$—in Table 3. In each of the two panels of Table 3, $Z_{j,k,\tau}$ is computed as the difference between the risk measure in the j-th row and the risk measures in the k-th column, for $j = 1, ..., 4$ and $k = 1, ..., 4$. We can also evaluate the trend of $Z_{j,k,\tau}$ over time in Figures 8—9 in Appendix D, where we average the proportions of the worsening only in the cross-sectional dimension, such that:

$$Z_{j,k,t,\tau} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\left\{\Delta Risk_{j,t,\tau}^{(i)} - \Delta Risk_{k,t,\tau}^{(i)} < -\phi\right\}}. \tag{32}$$

Table 3: Indicator $Z_{j,k,\tau}$

	$\tau = 0.01$			
	$\Delta CoVaR_\tau$	$\Delta CoCaViaR_\tau$	$\Delta QL\text{-}CoVaR_\tau$	$\Delta QL\text{-}CoCaViaR_\tau$
$\Delta CoVaR_\tau$	0.000	45.726	21.335	30.195
$\Delta CoCaViaR_\tau$	23.614	0.000	21.363	26.436
$\Delta QL\text{-}CoVaR_\tau$	71.861	72.503	0.000	54.264
$\Delta QL\text{-}CoCaViaR_\tau$	62.479	67.098	35.676	0.000
	$\tau = 0.05$			
	$\Delta CoVaR_\tau$	$\Delta CoCaViaR_\tau$	$\Delta QL\text{-}CoVaR_\tau$	$\Delta QL\text{-}CoCaViaR_\tau$
$\Delta CoVaR_\tau$	0.000	9.062	9.334	19.814
$\Delta CoCaViaR_\tau$	58.445	0.000	31.706	40.021
$\Delta QL\text{-}CoVaR_\tau$	73.947	47.930	0.000	44.645
$\Delta QL\text{-}CoCaViaR_\tau$	63.608	41.565	32.864	0.000

The table displays the values of $Z_{j,k,\tau}$ defined in (31) for each of the pairs of the risk measures ordered in the j-th row and in the k-th column, for $j = 1, ..., 4$ and $k = 1, ..., 4$.

When comparing $\Delta CoVaR_{t,\tau}^{(i)}$ and $\Delta CoCaViaR_{t,\tau}^{(i)}$, the sign of the worsening changes according to the value of τ. In fact, on average, $\Delta CoVaR_{t,\tau}^{(i)}$ provides a greater proportion of worsening with respect to $\Delta CoCaViaR_{t,\tau}^{(i)}$ at $\tau = 0.01$ (45.73% versus 23.61%; see Table 3). The opposite holds at $\tau = 0.05$—$\Delta CoCaViaR_{t,\tau}^{(i)}$ signals a larger worsening than $\Delta CoVaR_{t,\tau}^{(i)}$ (58.45% versus 9.06%). However, both $\Delta CoVaR_{t,\tau}^{(i)}$ and $\Delta CoCaViaR_{t,\tau}^{(i)}$ provide lower proportions of worsening in the systemic risk when compared with the quantile-located measures. In fact, the highest values of $Z_{j,k,\tau}$ in Table 3 are observed at $j = \{3, 4\}$, that is, when considering either ΔQL-CoVaR or ΔQL-CoCaViaR as $\Delta Risk^{(i)}_{j,t,\tau}$ in (31). This evidence is consistent with the results discussed above—the relationships between the system and the companies become more accentuated when focusing on the left tails of their distributions and the inclusion of the CaViaR's components absorbs, in part, the impact of the companies on the system. However, we stress that the empirical evidence is not pointing strictly dominating values of ΔQL-CoVaR (or ΔQL-CoCaViaR) with respect to $\Delta CoVaR$, as both $Z_{1,3,\tau}$ and $Z_{1,4,\tau}$ are greater than zero in Table 3.

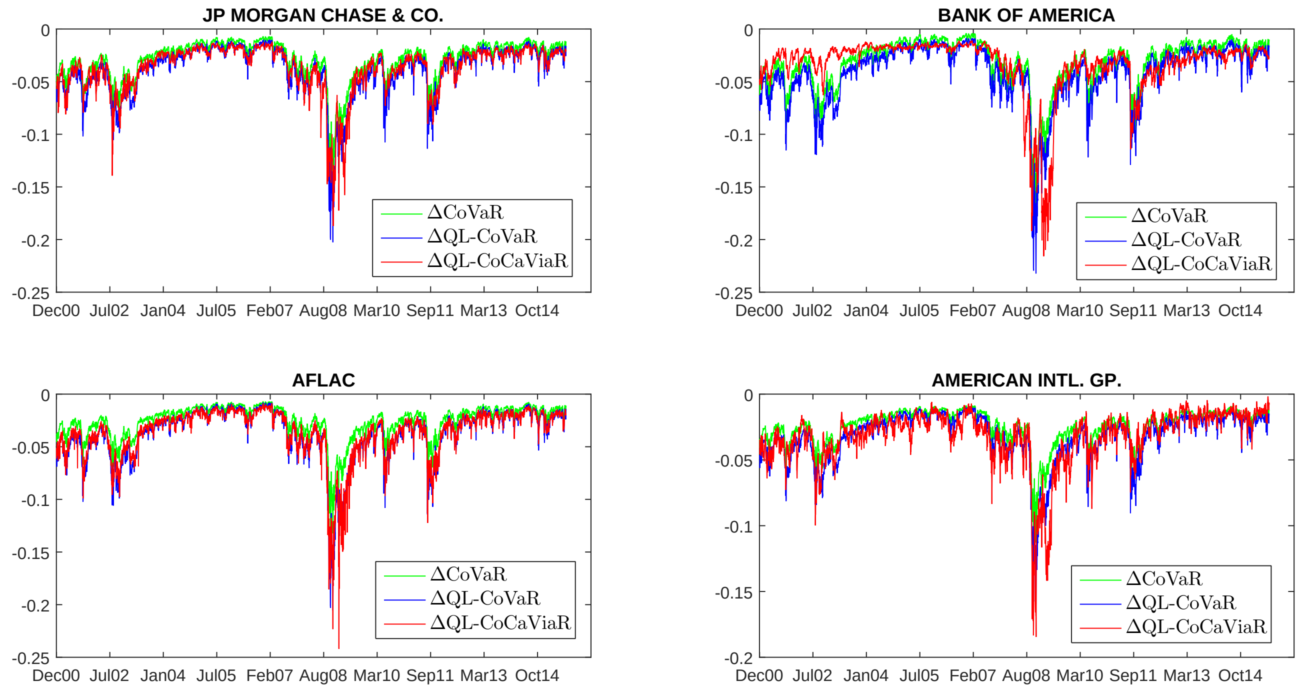

Figure 4: The four panels display the trend of ΔCoVaR, ΔQL-CoVaR and ΔQL-CoCaViaR for two selected banks and two selected insurance companies. The risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$.

Figure 4 provides an example for four selected companies: JP Morgan Chase & Co., Bank of America, Aflac and American International Group. Here, the quantile-located measures—ΔQL-CoVaR and ΔQL-CoCaViaR—almost always point out a greater risk with respect to ΔCoVaR. Nevertheless, the difference between the quantile-located measures and the ΔCoVaR is not constant over time. As described above, the quantile-located measures do not simply provide a linear downwards shift of risk with respect to the ΔCoVaR, but account for changes in the structural relations, especially during crisis periods. Besides, we can find sometimes periods in which the ΔCoVaR is greater (in absolute value) than the quantile-located measures. This opens the door for further analyses about the opportunity cost associated with the possible precautionary immobilization of financial resources to offset the impact of systemic events.

The interconnections among financial institutions become critical during special events, such as financial crises. Therefore, it is interesting to assess how the statistics of $\Delta CoVaR_{t,\tau}^{(i)}$, $\Delta CoCaViaR_{t,\tau}^{(i)}$, $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ and $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$ react to the occurrence of such events and, for this purpose, we consider the subprime crisis. Hence, we now compute the four risk measures from the data recorded in September 2008, reporting the descriptive statistics in Table 4. On average, the 5-th and the 95-th percentiles and the median of the four measures significantly change with respect to the full sample results, highlighting a greater contribution of the financial companies to the systemic risk during the subprime crisis. $\Delta QL\text{-}CoVaR_{t,\tau}^{(i)}$ and $\Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}$ are, on average, the most sensitive to the subprime crisis, recording the highest percentage variations in their medians with respect to the full sample results. We then have a further evidence that the quantiles-located relationships are critical during crisis periods, being important signaling tools that can be useful for preventing or mitigating the effects of extreme events.

Table 4: ΔCoVaR, ΔCoCaViaR, ΔQL-CoVaR and ΔQL-CoCaViaR during the subprime crisis.

	$\theta = \tau = 0.01$				$\theta = \tau = 0.05$			
	5P	MED	95P	IQR	5P	MED	95P	IQR
$\Delta CoVaR_{t,\tau}$	-2.232	-1.669	-1.209	0.524	-1.355	-1.004	-0.720	0.303
$\Delta CoCaViaR_{t,\tau}$	-1.776	-1.215	-0.781	0.490	-2.105	-1.445	-1.026	0.575
$\Delta QL\text{-}CoVaR_{t,\tau}$	-6.870	-4.890	-3.094	1.863	-3.820	-2.592	-1.623	1.120
$\Delta QL\text{-}CoCaViaR_{t,\tau}$	-6.440	-3.443	-1.540	2.565	-4.540	-2.879	-1.468	1.787

For the days in September 2008 and for each company of our dataset, we compute the four risk measures given in the first column at $\theta = \tau = \{0.01, 0.05\}$, setting $h = 0.15$ (the results obtained for other bandwidth values are similar and are available on request). For each company, we then compute the 5-th percentile (5P, %), the median (MED, %), the 95-th percentile (95P, %) and the interquartile range (IQR, %) of the four risk measures. Finally, we report, from left to right, the cross-sectional medians of the four descriptive statistics.

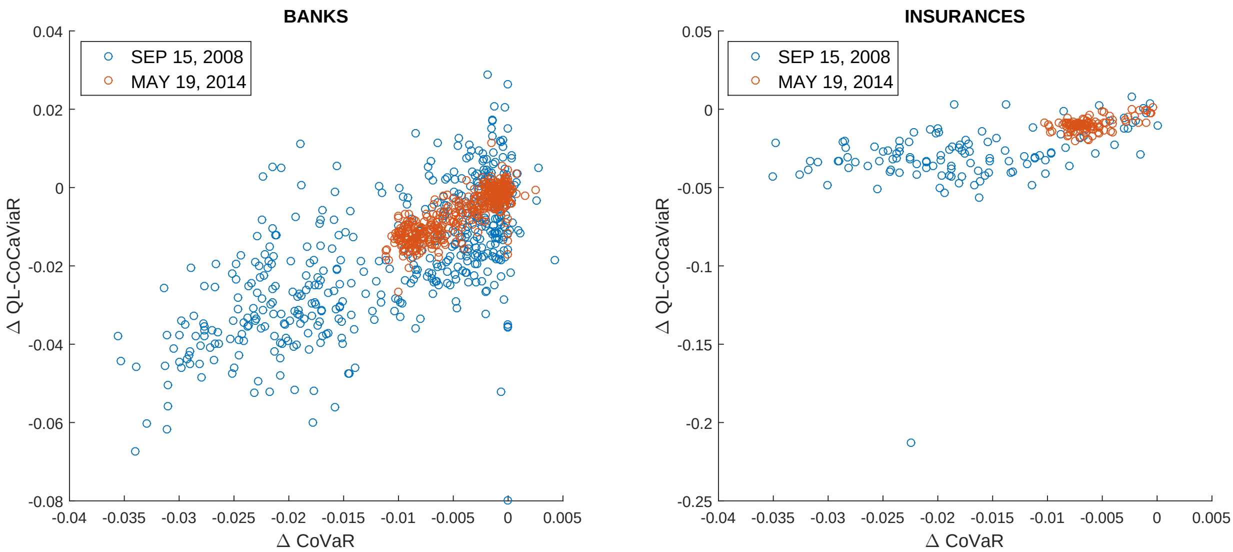

Figure 5: Scatter plots for banks and insurance companies of ΔCoVaR versus ΔQL-CoCaViaR on two specific days, that is, 15/09/2008 and 19/05/2014. The risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$.

Such a finding is also confirmed by the scatter plots in Figure 5, where we compare the ΔCoVaR with the ΔQL-CoCaViaR in a stressed (September 15, 2008) and in a quiet day (May 19, 2014).[10] Note that, if the difference between the two measures on the two different days were due to a volatility effect only, without any additional element coming from the covariates and the conditional quantile model structure, we would have not noted an increase in the scatter plot dispersion. In fact, the scatter plots would have displayed similar changes of the volatility (assuming all other elements play no role), providing a change of only the location and not of the scale. In the two panels of Figure 5 we observe, however, a marked increase in both the location of the scatter center and the dispersion, something that we cannot simply attribute to a change in the market risk over the two specific days. In addition, the relation between ΔCoVaR and ΔQL-CoCaViaR seems much stronger when the market is in a low volatility state. We link this to the possible structural changes in the relation between the financial market index, the financial companies and their corresponding quantiles, which might be observed during market turmoils. Moving from ΔCoVaR to ΔQL-CoCaViaR does not simply induce a linear shift of the systemic risk measures but account in a more proper way for the relation between the companies and the market when focusing on extreme market conditions.

We now decompose $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$ defined in (18) into the components $c^{(i)}_{j,t,\tau}$, for $j = 1, ..., 7$, as discussed in Section 2.5. We evaluate the relevance of the various elements we added to the standard ΔCoVaR by measuring the relative contribution of these components to $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$, the most general risk measure we developed. In particular, for each time t and for each i-th company, we compute the weights of the seven components as: $w^{(i)}_{1,t,\tau} = c^{(i)}_{1,\tau}/|\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}|$ and $w^{(i)}_{j,t,\tau} = c^{(i)}_{j,t,\tau}/|\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}|$, for $j = 2, ..., 7$. By considering the ratio between each ΔQL-CoCaViaR's determinant and the absolute value of $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$, we can compare, on the one hand, the magnitude of the weights of the seven components. On the other hand, we can asses the sign of their contributions—to understand whether, on average, each component moves the ΔQL-CoCaViaR leftwards or rightwards. For each company and for each time series of the weights, we compute the 5-th and the 95-th percentiles, the median and the interquartile range. Then, we report the cross-sectional medians of these statistics in Table 5. In addition, Figures 10—11 in Appendix D display the time evolution of the cross-sectional medians of the seven ratios.

We can see from Table 5 and Figures 10—11 that the second component of the ΔQL-CoCaViaR—the traditional ΔCoVaR revised under a quantile-located perspective—provides the most relevant contribution. Notably, $c^{(i)}_{2,t,\tau}$ is the core of $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$, as it measures the impact of the increment in the risk of the i-th company. We stress that the contribution of this component would have been less relevant without considering the quantile-located relations. In fact, the cross-sectional median of $\widehat{\lambda}^{(i)}_{\theta,\tau}$—entering both QL-CoVaR and QL-CoCaViaR—is greater, on average, than $\widehat{\lambda}^{(i)}_{\theta}$—entering both CoVaR and CoCaViaR, that is, the risk measures which do not include quantile-located ef-

[10] For simplicity we consider here only the comparison between ΔCoVaR and ΔQL-CoCaViaR. Similar results hold when comparing ΔCoVaR with ΔQL-CoVaR.

fects (see Section 4.1 and Appendix C). As a result, on average, the following inequality holds: $\left|\widehat{\lambda}^{(i)}_{\theta,\tau}\left[\widehat{Q}_\tau(x_{i,t})-\widehat{Q}_{1/2}(x_{i,t})\right]\right| > \left|\widehat{\lambda}^{(i)}_{\theta}\left[\widehat{Q}_\tau(x_{i,t})-\widehat{Q}_{1/2}(x_{i,t})\right]\right|$. It is interesting to observe that $w^{(i)}_{2,t,\tau}$—the weight of $c^{(i)}_{2,t,\tau}$—increases, on average, when switching from $\theta = \tau = 0.05$ to $\theta = \tau = 0.01$, that is, when increasing the distress degree in the financial connections, emphasizing the importance of the quantile-located effects. On average, $c^{(i)}_{2,t,\tau}$ takes negative values. This is an expected result, given that the coefficient $\lambda^{(i)}_{\theta,\tau}$ is positive on average (see Table 14), whereas $Q^{(i)}_\tau(x_{i,t}) - Q^{(i)}_{1/2}(x_{i,t}) < 0$.

The impact of the individual firms on the ΔQL-CoCaViaR depends also on $c^{(i)}_{3,t,\tau}$, that is, the median quantile of $x_{i,t}$—$Q_{1/2}(x_{i,t})$—times the residual impact of the i-th company when moving from the left tail to the center of the $x_{i,t}$'s distribution—$(\lambda^{(i)}_{\theta,\tau} - \lambda^{(i)}_{\theta,1/2})$. Nevertheless, the contribution of $c^{(i)}_{3,t,\tau}$ to the magnitude of $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$ is close to zero (see Table 5 and Figures 10—11). This is not surprising, as the median of the i-th company's returns—$Q_{1/2}(x_{i,t})$—is close, if not equal, to zero. Therefore, the impact of the individual companies to the systemic risk is entirely captured by $c^{(i)}_{2,t,\tau}$.

We checked that the persistence of the y_t's quantiles over time is affected by the co-movements between the system and the companies. Furthermore, the relevance of these co-movements changes according to the state in which the system and the individual companies are located (see Appendix C). In particular, on average, $\psi^{(i)}_{1,0.01,0.01}$ is less than half $\psi^{(i)}_{1,0.01,1/2}$, whereas the difference between $\psi^{(i)}_{1,0.05,0.05}$ and $\psi^{(i)}_{1,0.05,1/2}$ is almost imperceptible (see Table 14 in Appendix C). This result is reflected in $c^{(i)}_{6,t,\tau} = \left[\psi^{(i)}_{1,\theta,\tau} - \psi^{(i)}_{1,\theta,1/2}\right] Q^{(i)}_{\theta,\tau}(y_{t-1})$, whose contribution to $\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}$ is greater at $\theta = \tau = 0.01$—the worst scenario—than at $\theta = \tau = 0.05$.

Despite the relevant average value of $\psi^{(i)}_{1,\theta,1/2}$, mainly at $\theta = \tau = 0.01$ (see Table 14), the weight of $c^{(i)}_{7,t,\tau} = \psi^{(i)}_{1,\theta,1/2}\left[Q^{(i)}_{\theta,\tau}(y_{t-1}) - Q^{(i)}_{\theta,1/2}(y_{t-1})\right]$ is relatively low. Therefore, the lagged increase in the risk of the system (due to the worsening of the companies' performance) is less relevant than either the increase in the risk of the companies—$w^{(i)}_{2,t,\tau}$—or the persistence of the lagged quantiles of the system—$w^{(i)}_{6,t,\tau}$—when moving along the marginal distributions of the companies' returns.

We also highlight the weights of $c^{(i)}_{1,\tau}$, which is always negative and more relevant at $\theta = \tau = 0.01$, $c^{(i)}_{4,t,\tau}$ and $c^{(i)}_{5,t,\tau}$, which take, on average, positive values and do not significantly change between $\theta = \tau = 0.01$ and $\theta = \tau = 0.05$. We remind the reader that the last three components measure, respectively, the changes in location of the system's density ($c^{(i)}_{1,\tau}$), the changes in the impact of the control variables ($c^{(i)}_{4,t,\tau}$) and the changes in the relevance of the past returns of the system ($c^{(i)}_{5,t,\tau}$) when moving along the marginal distributions of the companies. Interestingly, the sign of the $c^{(i)}_{4,t,\tau}$'s contribution is not constant over time (see Figures 10—11). In particular, $c^{(i)}_{4,t,\tau}$ increases the systemic risk during market turmoils (the end of the technology market bubble, the 11th of September 2001, during the subprime crisis and the European sovereign crisis), while it reduces systemic risk when markets are upward trending (in particular during the years 2003—2006). This evidence suggests that the QL-CoCaViaR provides relevant insights on the role of control variables during turmoils. This is consistent with the interpretation of these events as systemic, given that the deterioration of the overall market conditions

Table 5: Statistics of the weights of the ΔQL-CoCaViaR's components

WEIGHTS	5P	MED	95P	IQR	5P	MED	95P	IQR
	$\theta = \tau = 0.01$, $h = 0.10$				$\theta = \tau = 0.05$, $h = 0.10$			
$w_{1,\tau}$	-101.856	-35.819	-11.609	95.065	-48.587	-16.754	-5.388	89.402
$w_{2,\tau}$	-131.081	-57.298	-26.414	41.808	-144.169	-54.247	-20.422	35.963
$w_{3,\tau}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$w_{4,\tau}$	-82.256	2.045	98.687	73.300	-72.392	3.351	95.588	73.746
$w_{5,\tau}$	0.084	4.283	21.818	23.833	0.032	1.513	7.336	20.949
$w_{6,\tau}$	1.116	22.366	56.510	75.889	-5.157	6.671	19.418	65.378
$w_{7,\tau}$	-61.971	-2.708	64.181	44.555	-70.861	-1.603	76.039	40.945
	$\theta = \tau = 0.01$, $h = 0.15$				$\theta = \tau = 0.05$, $h = 0.15$			
$w_{1,\tau}$	-82.921	-29.271	-10.113	78.493	-25.660	-10.464	-3.313	72.051
$w_{2,\tau}$	-134.561	-67.777	-33.145	40.591	-143.669	-62.208	-24.930	34.405
$w_{3,\tau}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$w_{4,\tau}$	-72.173	2.344	86.350	67.441	-54.310	2.847	74.098	56.549
$w_{5,\tau}$	0.076	3.487	16.212	19.922	0.035	1.443	6.146	18.625
$w_{6,\tau}$	1.970	22.110	54.088	64.325	-4.347	3.583	11.440	53.354
$w_{7,\tau}$	-53.781	-1.309	63.991	42.919	-61.939	-1.295	60.295	33.211
	$\theta = \tau = 0.01$, $h = 0.20$				$\theta = \tau = 0.05$, $h = 0.20$			
$w_{1,\tau}$	-33.028	-15.017	-5.663	61.374	-14.577	-6.544	-2.239	48.040
$w_{2,\tau}$	-134.641	-81.957	-41.311	38.337	-146.176	-75.290	-33.567	32.389
$w_{3,\tau}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$w_{4,\tau}$	-56.540	1.211	70.667	55.939	-39.682	2.687	61.254	44.534
$w_{5,\tau}$	0.064	3.125	14.809	15.729	0.032	1.473	5.815	16.004
$w_{6,\tau}$	0.630	13.963	30.293	54.516	-2.641	3.148	8.471	39.197
$w_{7,\tau}$	-48.770	-0.677	54.998	38.114	-52.468	-0.598	42.306	25.688

For each day and for each company, we compute the weights of the ΔQL-CoCaViaR's components. For each company and for each time series of the weights, we then compute the following statistics: the 5-th (5P, %) and the 95-th percentiles (95P, %), the median (MED, %) and the interquartile range (IQR, %). The table reports the cross-sectional medians of these statistics.

might further contribute to the system risk through a set of control or state variables.

Summing up, the findings discussed above highlight the importance of the additional elements added to the standard ΔCoVaR. The new components take into account the persistence of the conditional quantiles over time (CoCaViaR), the quantile-located effects (QL-CoVaR) and their combination (QL-CoCaViaR).

4.3 Backtesting results

The out-of-sample accuracy of $CoVaR^{(i)}_{t,\theta,\tau}$, $CoCaViaR^{(i)}_{t,\theta,\tau}$, $QL\text{-}CoVaR^{(i)}_{t,\theta,\tau}$ and $QL\text{-}CoCaViaR^{(i)}_{t,\theta,\tau}$ is evaluated using the backtesting methods described in Section 2.6. In particular, we implement the rolling window procedure with $ws = 300$. As for the estimates, we set $\theta = \tau = 0.05$ and $h = 0.15$.[11] First, we compare the predictive accuracy of the four risk measures using the unconditional and the conditional coverage tests, without conditioning the hit of the system to the distress state of the individual financial companies. Table 6 reports the percentages of times (over time and over the cross-

[11]The results obtained with other values of θ, τ and h are available upon request.

Table 6: Unconditional and conditional coverage tests

	UNCONDITIONAL COVERAGE TEST				CONDITIONAL COVERAGE TEST			
NOBS	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR
100	38.422	43.511	32.353	39.386	38.931	43.003	31.714	34.015
200	36.063	41.876	30.000	37.467	36.988	41.347	29.200	31.600
300	32.587	38.741	26.346	34.703	33.427	38.322	25.354	28.329
400	30.891	37.213	25.109	33.527	31.753	36.782	24.093	27.141

For each of the four risk measures, that is, CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR, the table reports the percentages of times over the cross-section and over time in which the null hypothesis of the unconditional coverage test by Kupiec (1995) and the null hypothesis of the conditional coverage test by Christoffersen (1998) are not rejected at the 1% significance level, when we have at least *NOBS* number of observations for each series in (25). CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR are computed by setting $\theta = \tau = 0.05$ and $h = 0.15$.

section) in which we do not reject the null hypothesis of the tests at the 1% significance level. Given the features of our dataset, we implement the tests on the series for which we have at lest 100, 200, 300 and 400 observations for the hit functions. We note that $CoCaViaR^{(i)}_{t,0.05,0.05}$ records the highest percentages of non-rejection in all cases.

Table 7: Unconditional and conditional coverage tests during the subprime crisis

	UNCONDITIONAL COVERAGE TEST				CONDITIONAL COVERAGE TEST			
PERIOD	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR
Jul 08-Sep 08	47.395	65.509	60.150	69.424	47.643	66.501	60.652	72.431
Jul 08-Oct 08	62.879	81.061	75.255	76.786	62.879	82.323	76.276	80.612
Jul 08-Nov 08	70.229	75.064	84.062	73.779	70.738	78.626	85.090	75.578
Jul 08-Dec 08	84.103	84.872	89.637	78.497	84.872	90.769	91.710	83.161

For each of the four risk measures, that is, CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR, the table reports the percentages of times over the cross-section and over time in which the null hypothesis of the unconditional coverage test by Kupiec (1995) and the null hypothesis of the conditional coverage test by Christoffersen (1998) are not rejected at the 1% significance level. The tests are implemented using four different periods: 01/07/2008—30/09/2008, 01/07/2008—31/10/2008, 01/07/2008—30/11/2008 and 01/07/2008—31/12/2008. CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR are computed by setting $\theta = \tau = 0.05$ and $h = 0.15$.

We now repeat the same exercise by implementing the Kupiec (1995) and the Christoffersen (1998) tests during the subprime crisis, to check whether the performance of the four risk measures changes at the distress state of the financial system. Table 7 report the results in four different time intervals, differing in their endpoint: 01/07/2008-30/09/2008, 01/07/2008-31/10/2008, 01/07/2008-30/11/2008 and 01/07/2008-31/12/2008. We observe that $QL\text{-}CoCaViaR^{(i)}_{t,0.05,0.05}$ records the best performance for both the unconditional and the conditional coverage tests when we use the shortest interval, that is, 01/07/2008—30/09/2008. Differently, $CoCaViaR^{(i)}_{t,0.05,0.05}$ records the highest percentage of non-rejections in the interval 01/07/2008—31/10/2008 whereas $QL\text{-}CoVaR^{(i)}_{t,0.05,0.05}$ over-performs the other competitive measures in the remaining periods.

So far we did not take into account the distress state of the conditioning company. In contrast, Table 8 reports the results we obtain by conditioning the system's hit function to the distress state

Table 8: Unconditional and conditional coverage tests at the distress state of the conditioning companies

	UNCONDITIONAL COVERAGE TEST				CONDITIONAL COVERAGE TEST			
PERIOD	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR
GE-1	28.172	30.125	27.736	29.910	28.731	28.877	34.528	25.946
GE-2	54.225	47.042	66.857	35.286	57.324	48.732	70.429	37.571
GE-3 (0.10)	76.852	62.943	94.947	51.323	81.217	67.674	95.213	55.556
GE-3 (0.20)	61.477	51.406	83.062	39.865	64.966	54.083	85.366	42.432
GE-3 (0.30)	52.162	45.202	65.439	35.684	55.649	46.453	68.130	38.223

For each of the four risk measures, that is, CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR, the table reports the percentages of times over the cross-section and over time in which the null hypothesis of the unconditional coverage test by Kupiec (1995) and the null hypothesis of the conditional coverage test by Christoffersen (1998) are not rejected at the 1% significance level. The tests are implemented during the periods of distress state of the conditioning financial companies. We detect the distress state of the financial companies using the hit functions (26) for GE-1 and (27) for GE-2, whereas we compute the hit function (28) at 3 different threshold values ths—0.10 (GE-3 (0.10)), 0.20 (GE-3 (0.20)) and 0.30 (GE-3 (0.30)). We implement the tests conditional to the fact that we have at least 50 days in which a given financial company is in a distress state, that is, 50 ones for each hit series of the company. CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR are computed by setting $\theta = \tau = 0.05$ and $h = 0.15$.

of each financial company. We detect the distress state of the individual financial companies using different hit functions for $x_{i,t}$. The percentages of non-rejections we have when using (26) and (27) are given in GE-1 and in GE-2, respectively. In contrast, we compute (28) at three different threshold values ν for a sensitivity analysis—$\nu = 0.10$ in GE-3 (0.10), $\nu = 0.20$ in GE-3 (0.20) and $\nu = 0.30$ in GE-3 (0.30). For each of the hit functions in (26)—(28), we implement the unconditional and the conditional coverage tests for y_t conditional to the fact that we have at least 50 days in which $Hit^i_{t,0.5,0.5} = 1$, that is, at least 50 days in which the i-th financial company is in a distress state.

Conditional to the distress state of the financial companies, in general, $QL\text{-}CoVaR^{(i)}_{t,0.05,0.05}$ over-performs the other risk measures. Notably, the percentages of non-rejections for the quantile-located measures are greater in GE-3, as the hit function of the conditioning companies in (28) is more consistent to the method we use to estimate both $QL\text{-}CoVaR^{(i)}_{t,0.05,0.05}$ and $QL\text{-}CoCaViaR^{(i)}_{t,0.05,0.05}$.

Table 9: Average loss in different periods

PERIOD	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR
01/01/2004 - 02/07/2004	4.972	7.075	4.977	7.339
01/07/2008 - 31/12/2008	10.063	9.653	9.344	9.762
01/01/2015 - 03/07/2015	4.297	4.213	3.740	4.611

The table reports the means over the cross-section of the losses coming from the CoVaR, the CoCaViaR, the QL-CoVaR and the QL-CoCaViaR in three different periods: 01/01/2004-02/07/2004, 01/07/2008-31/12/2008 and 01/01/2015-03/07/2015. CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR are computed by setting $\theta = \tau = 0.05$ and $h = 0.15$.

We report in Table 9 the means over i of the losses $LOSS^{CoVaR|i}_{0.05,0.05}$, $LOSS^{CoCaViaR|i}_{0.05,0.05}$, $LOSS^{QL\text{-}CoVaR|i}_{0.05,0.05}$ and $LOSS^{QL\text{-}CoCaViaR|i}_{0.05,0.05}$ (see Section 2.6) for 3 different periods having the same size of 132 observations, that is, the pre–subprime crisis period (01/01/2004—02/07/2004), the subprime crisis period (01/07/2008—31/12/2008) and the post–subprime crisis period (01/01/2015—03/07/2015). As expected, the losses are smaller during tranquil periods and considerably increase during the subprime crisis. On average, the CoVaR overperforms the competitive measures in the interval 01/01/2004—02/07/2004, whereas the QL-CoVaR provides the lowest loss in the other periods. Interestingly, the distance between $CoVaR^{(i)}_{t,0.05,0.05}$ and $QL\text{-}CoVaR^{(i)}_{t,0.05,0.05}$ increases in periods of financial distress, such as the days in the interval 01/07/2008—31/12/2008.

Table 10: Difference in the performance statistically validated

	CoVaR	CoCaViaR	QL-CoVaR	QL-CoCaViaR
01/01/2004-02/07/2004				
CoVaR	0.000	21.622	11.824	31.757
CoCaViaR	1.014	0.000	3.378	18.243
QL-CoVaR	9.797	29.054	0.000	29.392
QL-CoCaViaR	1.689	15.541	1.014	0.000
01/07/2008-31/12/2008				
CoVaR	0.000	6.826	3.754	5.802
CoCaViaR	7.850	0.000	4.778	9.215
QL-CoVaR	22.867	14.334	0.000	8.532
QL-CoCaViaR	9.215	11.263	2.389	0.000
01/01/2015-03/07/2015				
CoVaR	0.000	7.071	3.030	9.091
CoCaViaR	2.694	0.000	4.040	8.418
QL-CoVaR	4.377	8.754	0.000	9.764
QL-CoCaViaR	2.020	6.061	0.000	0.000

For each of the three periods: 01/01/2004—02/07/2004, 01/07/2008—31/12/2008 and 01/01/2015—03/07/2015, the table reports the percentages of times over the cross-section in which the risk measure in the i-th row over-performs the risk measure in the j-th column for each of 3 panels, for $i, j = 1, ..., 4$. The different performances of the four risk measures, that is, CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR, are validated by the Diebold and Mariano (2002) test at the 0.05 significance level. CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR are computed by setting $\theta = \tau = 0.05$ and $h = 0.15$.

We also test whether the losses defined in (30) generated by the four risk measures—CoVaR, CoCaViaR, QL-CoVaR and QL-CoCaViaR—are statistically different. For this purpose, we implement the test proposed by Diebold and Mariano (2002) at the 5% significance level. For each of the three panels in Table 10, which respectively refer to the periods: 01/01/2004—02/07/2004, 01/07/2008—31/12/2008 and 01/01/2015—03/07/2015, we report the percentages of times over the cross-section in which the risk measure in the row outperforms, is a statistically significant way, the risk measure in the column. For instance, the CoVaR outperforms the CoCaViaR in 21.62% of the cases during the period 01/01/2004—02/07/2004, whereas the CoCaViaR outperforms the CoVaR in 1.01% of the cases in the same period. It is interesting to see that when considering the net values, that is, the percentage of times in which a given measure outperforms another measure minus the percentage of times in which it is outperformed by the same competitor, the quantile-located measures

(QL-CoVaR and QL-CoCaViaR) record the best results during the crisis period. Indeed, in the period 01/07/2008—31/12/2008 the gap between the quantile-located measures and the other competitors is positive and tends to be greater with respect to the other periods.

To summarize, we assess the predictive accuracy of the risk measures by means of different backtesting methods, based on different hit and loss functions, and compare the results obtained in calm and crisis periods, or conditioning the system to the distress state of the individual financial companies. We verify that one of the quantile-located risk measures, that is, the QL-CoVaR, outperforms the standard CoVaR and the other extensions we propose when we increase the distress degree in the connections between the financial system and the individual companies. Consistent with the in-sample results (see the comparison between $\mathit{QL\text{-}CoVaR}^{(i)}_{t,\theta,\tau}$ and $\mathit{QL\text{-}CoCaViaR}^{(i)}_{t,\theta,\tau}$ in Section 4.2), the inclusion of the CaViaR autoregressive components lowers the sensitivity of y_t to $x_{i,t}$ and compromises the predictive accuracy of the quantile-located method when we link the lower quantiles of both the system and the conditioning companies.

5 Concluding remarks

We extend the CoVaR introduced by Adrian and Brunnermeier (2016), taking into account the persistence of the conditional quantiles over time (CoCaViaR), the quantile-located relationships (QL-CoVaR) and their combination (QL-CoCaViaR). An extensive empirical analysis based on a large dataset including U.S. banks and insurance companies highlights the relevance of the new elements we introduce. First, we checked that the CoCaViaR is more conservative than the other risk measures as the CaViaR's components (Engle and Manganelli, 2004) absorb, in part, the relations between the financial system and the individual companies, lowering the contribution of the firms to the systemic risk. Second, the relationships between the financial system and the individual companies become stronger when linking the left tails of their returns' distributions, that is, when accentuating the distress degree in their connections (quantile-located effects). As a result, our approach is more sensitive to crisis periods. The in-sample results are confirmed out-of-sample. In particular, we use various backtesting methods for evaluating the predictive accuracy of the competitive risk measures. Some of them are well-known in the literature, while others are new being developed to deal with the quantile-located method we propose here. In particular, the empirical evidence shows that the quantile-located relationships are particularly useful in improving the predictive accuracy during stressed periods, making it a relevant tool for financial regulators for managing or preventing the effects of extreme events.

References

Adrian, T. and M. K. Brunnermeier (2016). Covar. *American Economic Review 106*, 1705–1741.

Bernal, O., J. Gnabo, and G. Guilmin (2014). Assessing the contribution of banks, insurance and other financial services to systemic risk. *Journal of Banking & Finance 47*, 270–287.

Bernardi, M., G. Gayraud, and L. Petrella (2015). Bayesian tail risk interdependence using quantile regression. *Bayesian Analysis 10*, 553–603.

Bernardi, M., A. Maruotti, and L. Petrella (2017). Multiple risk measures for multivariate dynamic heavy–tailed models. *Journal of Empirical Finance 43*, 1–32.

Bondell, H. D., B. J. Reich, and H. Wang (2010). Non-crossing quantile regression curve estimation. *Biometrika 97*, 825–838.

Caporin, M. (2008). Evaluating value-at-risk measures in the presence of long memory conditional volatility. *Journal of Risk 10*, 79–110.

Castro, C. and S. Ferrari (2014). Measuring and testing for the systemically important financial institutions. *Journal of Empirical Finance 25*, 1–14.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review 39*, 841–862.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance 1*(2), 223–236.

Diebold, F. and R. Mariano (2002). Comparing predictive accuracy. *Journal of Business & Economic Statistics 20*, 134–144.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics 7*, 1–26.

Engle, R. F. and S. Manganelli (2004). Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics, American Statistical Association 22*, 367–381.

Girardi, G. and A. T. Ergun (2013). Systemic risk measurement: Multivariate GARCH estimation of CoVaR. *Journal of Banking and Finance 37*, 3169–3180.

Kocherginsky, M. (2003). *Extensions of the Markov chain marginal bootstrap.* PhD Thesis, University of Illinois Urbana-Champaign.

Koenker, R. (2005). *Quantile regression.* Number 38. Cambridge university press.

Koenker, R. and G. Bassett (1978). Regression quantiles. *Econometrica 46*(1), 33–50.

Koenker, R. and Z. Xiao (2004). Unit root quantile autoregression inference. *Journal of the American Statistical Association 99*, 775–787.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives 3*, 73–84.

Li, G., Y. Li, and C. L. Tsai (2015). Quantile correlations and quantile autoregressive modeling. *Journal of the American Statistical Association 110*, 246–261.

López-Espinosa, G., A. Moreno, A. Rubia, and L. Valderrama (2012). Short-term wholesale funding and systemic risk: A global covar approach. *Journal of Banking & Finance 36*, 3150–3162.

López-Espinosa, G., A. Moreno, A. Rubia, and L. Valderrama (2015). Systemic risk and asymmetric responses in the financial industry. *Journal of Banking & Finance 58*, 471–485.

Qu, Z. (2008). Testing for structural change in regression quantiles. *Journal of Econometrics 146*, 170–184.

Sim, N. and H. Zhou (2015). Oil prices, us stock return, and the dependence between their quantiles. *Journal of Banking & Finance 55*, 1–8.

White, H., T. Kim, and S. Manganelli (2015). Var for var: Measuring tail dependence using multivariate regression quantiles. *Journal of Econometrics 187*, 169–188.

APPENDIX

A Derivation of the Δ*QL-CoVaR* in Equation (14)

$$\begin{aligned}
\Delta QL\text{-}CoVaR^{(i)}_{t,\tau} &= QL\text{-}CoVaR^{(i)}_{t,\theta,\tau} - QL\text{-}CoVaR^{(i)}_{t,\theta,1/2} \\
&= \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1} - \left[\widehat{\delta}^{(i)}_{\theta,1/2} + \widehat{\lambda}^{(i)}_{\theta,1/2}\widehat{Q}_{1/2}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2}\mathbf{M}'_{t-1}\right] \\
&= \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1} - \left[\widehat{\delta}^{(i)}_{\theta,1/2} + \widehat{\lambda}^{(i)}_{\theta,1/2}\widehat{Q}_{1/2}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2}\mathbf{M}'_{t-1}\right] \\
&+ \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{1/2}(x_{i,t}) - \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{1/2}(x_{i,t}) \\
&= \left(\widehat{\delta}^{(i)}_{\theta,\tau} - \widehat{\delta}^{(i)}_{\theta,1/2}\right) + \widehat{\lambda}^{(i)}_{\theta,\tau}\left[\widehat{Q}_{\tau}(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t})\right] + (\widehat{\lambda}^{(i)}_{\theta,\tau} - \widehat{\lambda}^{(i)}_{\theta,1/2})\widehat{Q}_{1/2}(x_{i,t}) \\
&+ (\widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau} - \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2})\mathbf{M}'_{t-1}
\end{aligned}$$

B Derivation of the $\Delta QL\text{-}CoCaViaR$ in Equation (18)

$$
\begin{aligned}
\Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau} &= QL\text{-}CoCaViaR^{(i)}_{t,\theta,\tau} - QL\text{-}CoCaViaR^{(i)}_{t,\theta,1/2} \\
&= \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\psi}^{(i)}_{1,\theta,\tau}\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) + \widehat{\psi}^{(i)}_{2,\theta,\tau}|y_{t-1}| + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1} \\
&- \widehat{\delta}^{(i)}_{\theta,1/2} - \widehat{\psi}^{(i)}_{1,\theta,1/2}\widehat{Q}^{(i)}_{\theta,1/2}(y_{t-1}) - \widehat{\psi}^{(i)}_{2,\theta,1/2}|y_{t-1}| - \widehat{\lambda}^{(i)}_{\theta,1/2}\widehat{Q}_{1/2}(x_{i,t}) \\
&- \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2}\mathbf{M}'_{t-1} \\
&= \widehat{\delta}^{(i)}_{\theta,\tau} + \widehat{\psi}^{(i)}_{1,\theta,\tau}\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) + \widehat{\psi}^{(i)}_{2,\theta,\tau}|y_{t-1}| + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{\tau}(x_{i,t}) + \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau}\mathbf{M}'_{t-1} \\
&- \widehat{\delta}^{(i)}_{\theta,1/2} - \widehat{\psi}^{(i)}_{1,\theta,1/2}\widehat{Q}^{(i)}_{\theta,1/2}(y_{t-1}) - \widehat{\psi}^{(i)}_{2,\theta,1/2}|y_{t-1}| - \widehat{\lambda}^{(i)}_{\theta,1/2}\widehat{Q}_{1/2}(x_{i,t}) \\
&- \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2}\mathbf{M}'_{t-1} + \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{1/2}(x_{i,t}) - \widehat{\lambda}^{(i)}_{\theta,\tau}\widehat{Q}_{1/2}(x_{i,t}) \\
&+ \widehat{\psi}^{(i)}_{1,\theta,1/2}\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) - \widehat{\psi}^{(i)}_{1,\theta,1/2}\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) \\
&= \left(\widehat{\delta}^{(i)}_{\theta,\tau} - \widehat{\delta}^{(i)}_{\theta,1/2}\right) + \widehat{\lambda}^{(i)}_{\theta,\tau}\left[\widehat{Q}_{\tau}(x_{i,t}) - \widehat{Q}_{1/2}(x_{i,t})\right] + (\widehat{\lambda}^{(i)}_{\theta,\tau} - \widehat{\lambda}^{(i)}_{\theta,1/2})\widehat{Q}_{1/2}(x_{i,t}) \\
&+ (\widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,\tau} - \widehat{\boldsymbol{\gamma}}^{(i)}_{\theta,1/2})\mathbf{M}'_{t-1} + (\widehat{\psi}^{(i)}_{2,\theta,\tau} - \widehat{\psi}^{(i)}_{2,\theta,1/2})|y_{t-1}| \\
&+ \left[\widehat{\psi}^{(i)}_{1,\theta,\tau} - \widehat{\psi}^{(i)}_{1,\theta,1/2}\right]\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) \\
&+ \widehat{\psi}^{(i)}_{1,\theta,1/2}\left[\widehat{Q}^{(i)}_{\theta,\tau}(y_{t-1}) - \widehat{Q}^{(i)}_{\theta,1/2}(y_{t-1})\right]
\end{aligned}
$$

C Analysis of the risk measures' coefficients

Table 11: Estimation of $Q^{(i)}_{\theta}(y_t) = \delta^{(i)}_{\theta} + \lambda^{(i)}_{\theta}x_{i,t} + \gamma^{(i)}_{\theta}fpc_{t-1}$

	$\theta = 0.01$					$\theta = 0.05$				
COEF	5P	MED	95P	IQR	PS	5P	MED	95P	IQR	PS
δ_{θ}	-0.042	-0.031	-0.021	0.012	99.903	-0.027	-0.019	-0.014	0.007	99.612
λ_{θ}	-0.033	0.117	0.536	0.236	45.146	-0.006	0.112	0.561	0.276	57.087
$100 \times \gamma_{\theta}$	-0.295	-0.197	-0.074	0.107	88.447	-0.202	-0.128	-0.079	0.064	95.243

The table reports the summary statistics of the CoVaR's parameters estimated for the N financial companies included in our dataset. The estimates are obtained using two quantile levels—θ. In each panel, from left to right, we report the following descriptive statistics of the coefficients: the 5-th percentile (5P), the median (MED), the 95-th percentile (3Q), the interquartile range (IQR) and the percentage of times, out of N, in which they are statistically significant at the 5% confidence level (PS).

Table 11 reports the statistics of the CoVaR's coefficients, obtained by estimating $Q^{(i)}_{\theta}(y_t) = \delta^{(i)}_{\theta} + \lambda^{(i)}_{\theta}x_{i,t} + \gamma^{(i)}_{\theta}fpc_{t-1}$, for $i = 1, ..., N$. On average, the financial companies have a positive impact on $Q^{(i)}_{\theta}(y_t)$. Indeed, the medians of $\widehat{\lambda}^{(i)}_{0.01}$ and $\widehat{\lambda}^{(i)}_{0.05}$—the coefficients monitoring the impact of a financial company on the market risk at the 1% and 5% confidence levels—are similar, being equal to 0.117 and 0.112, respectively. $\widehat{\lambda}^{(i)}_{0.05}$ is slightly more volatile than $\widehat{\lambda}^{(i)}_{0.01}$, with an interquartile range of 0.276 (versus 0.236). Both the 5-th and the 95-th percentiles are larger for $\widehat{\lambda}^{(i)}_{0.05}$ (-0.006 and 0.561, respectively) than for $\widehat{\lambda}^{(i)}_{0.01}$ (-0.033 and 0.536, respectively). $\widehat{\lambda}^{(i)}_{0.05}$ records a larger number of times in which it is statistically significant at the 5% confidence level with respect to $\widehat{\lambda}^{(i)}_{0.01}$ (57% versus 45%). Overall, we do not observe relevant differences in the impact of financial companies on the system when comparing $\theta = 0.05$ and $\theta = 0.01$. When focusing on the impact of the control variables, as expressed by their first principal component (fpc_{t-1}), we again observe similarities when comparing $\theta = 0.01$ and $\theta = 0.05$. We first note that the low values of $\widehat{\gamma}_{\theta}^{(i)}$ are due to the scale in which fpc_{t-1} is expressed (see Section 3). Notably, $\widehat{\gamma}_{\theta}^{(i)}$ is statistically significant for almost all the N companies, highlighting a relevant role of the control variables in the evaluation of the market risk.

Table 12: Estimation of $Q_{\theta}^{(i)}(y_t) = \delta_{\theta}^{(i)} + \psi_{1,\theta}^{(i)} Q_{\theta}^{(i)}(y_{t-1}) + \psi_{2,\theta}^{(i)}|y_{t-1}| + \lambda_{\theta}^{(i)} x_{i,t} + \gamma_{\theta}^{(i)} fpc_{t-1}$

	$\theta = 0.01$					$\theta = 0.05$				
COEF	5P	MED	95P	IQR	PS	5P	MED	95P	IQR	PS
δ_{θ}	-0.042	-0.022	-0.001	0.020	78.155	-0.027	-0.014	-0.001	0.015	83.689
$\psi_{1,\theta}$	-0.491	0.197	0.917	0.916	55.728	-0.616	0.142	0.895	0.815	52.816
$\psi_{2,\theta}$	-0.321	-0.132	0.147	0.203	38.932	-0.211	-0.100	0.067	0.134	42.427
λ_{θ}	-3.711	0.101	0.510	0.248	78.447	-0.005	0.097	0.557	0.280	76.505
$100 \times \gamma_{\theta}$	-0.251	-0.119	-0.003	0.161	70.194	-0.159	-0.083	-0.005	0.096	82.524

The table reports the summary statistics of the CoCaViaR's parameters for the N financial companies included in our dataset. The estimates are obtained using two quantile levels—θ. In each panel, from left to right, we report the following descriptive statistics of the coefficients: the 5-th percentile (5P), the median (MED), the 95-th percentile (95P), the interquartile range (IQR) and the percentage of times, out of N, in which they are statistically significant at the 5% confidence level (PS).

Table 12 includes the descriptive statistics of the CoCaViaR's coefficients estimated from $Q_{\theta}^{(i)}(y_t) = \delta_{\theta}^{(i)} + \psi_{1,\theta}^{(i)} Q_{\theta}^{(i)}(y_{t-1}) + \psi_{2,\theta}^{(i)}|y_{t-1}| + \lambda_{\theta}^{(i)} x_{i,t} + \gamma_{\theta}^{(i)} fpc_{t-1}$. On average, the lagged financial system quantile $Q_{\theta}^{(i)}(y_{t-1})$ has a positive impact on the current financial system quantile, $Q_{\theta}^{(i)}(y_t)$, and the relationships are stronger at $\theta = 0.01$ (the median of $\widehat{\psi}_{1,0.01}^{(i)}$ is equal to 0.197) than $\theta = 0.05$ (the median of $\widehat{\psi}_{1,0.05}^{(i)}$ is equal to 0.142). This signals that the persistence in the quantiles is more relevant in more extreme market states. We also point out that $\widehat{\psi}_{1,0.01}^{(i)}$ and $\widehat{\psi}_{1,0.05}^{(i)}$ take moderate values, lower than the levels typically observed in the CaViaR model by Engle and Manganelli (2004). Therefore, the inclusion of the dependence of the system on a financial company absorb, in part, the effects typically exerted by the CaViaR components. $\widehat{\psi}_{1,0.01}^{(i)}$ is slightly more volatile than $\widehat{\psi}_{1,0.05}^{(i)}$, as the interquartile range of the former is equal to 0.916 (versus 0.816). Further, $\widehat{\psi}_{1,0.01}^{(i)}$ and $\widehat{\psi}_{1,0.05}^{(i)}$ are statistically significant at the 0.05 level for more than half of the N companies.

On average, $|y_{t-1}|$, the symmetric innovation term of the CoCaViaR, has a negative impact on $Q_{\theta}^{(i)}(y_t)$: the greater the absolute value of the lagged returns of the system the higher is the extreme risk of the system. Therefore, a very bad or a very good day symmetrically increase the probability of observing greater losses in the next day, consistent with the hypotheses laid down by Engle and Manganelli (2004). Analyzing the coefficients associated with the symmetric innovation, we first note that $\widehat{\psi}_{2,0.01}^{(i)}$ is slightly more volatile than $\widehat{\psi}_{2,0.05}^{(i)}$, as its interquartile range is equal to 0.203 (versus 0.133). In addition, $\widehat{\psi}_{2,\theta}^{(i)}$ records a lower percentage of times in which it is statistically significant with respect to the other CaViaR's parameter, that is $\widehat{\psi}_{1,\theta}^{(i)}$. Moving now to the loadings of both the financial company's returns and the conditioning variables' first principal component, we observe that the statistics of $\widehat{\lambda}_{\theta}^{(i)}$ and of $\widehat{\gamma}_{\theta}^{(i)}$ slightly change when switching from the CoVaR to the CoCaViaR. For the CoCaViaR financial companies returns positively impact (in median) on the system, with a somewhat larger effect at the 1% quantile. fpc_{t-1} is statistically relevant in a large fraction of cases. However, for both $\widehat{\lambda}_{\theta}^{(i)}$ and of $\widehat{\gamma}_{\theta}^{(i)}$ we observe a contraction in the estimated values, when moving from CoVaR to CoCaViaR. This is most likely an effect due to the presence of a quantile autoregressive component.

Table 13: Estimation of $Q_{\theta,\tau}^{(i)}(y_t) = \delta_{\theta,\tau}^{(i)} + \lambda_{\theta,\tau}^{(i)} x_{i,t} + \gamma_{\theta,\tau}^{(i)} fpc_{t-1}$ and $Q_{\theta,1/2}^{(i)}(y_t) = \delta_{\theta,1/2}^{(i)} + \lambda_{\theta,1/2}^{(i)} x_{i,t} + \gamma_{\theta,1/2}^{(i)} fpc_{t-1}$

COEF	5P	MED	95P	IQR	PS	5P	MED	95P	IQR	PS
	$\theta = \tau = 0.01,\ h = 0.10$					$\theta = \tau = 0.05,\ h = 0.10$				
$\delta_{\theta,\tau}$	-0.050	-0.029	-0.015	0.015	91.262	-0.031	-0.019	-0.010	0.008	94.175
$\delta_{\theta,1/2}$	-0.041	-0.028	-0.017	0.012	97.864	-0.025	-0.017	-0.011	0.006	98.447
$\lambda_{\theta,\tau}$	-0.213	0.218	0.980	0.518	24.660	-0.089	0.205	0.884	0.471	35.728
$\lambda_{\theta,1/2}$	-0.470	0.349	1.135	0.588	32.427	-0.199	0.354	0.910	0.453	46.019
$100 \times \gamma_{\theta,\tau}$	-0.392	-0.193	0.002	0.155	72.816	-0.258	-0.146	-0.042	0.080	84.563
$100 \times \gamma_{\theta,1/2}$	-0.302	-0.176	-0.037	0.102	84.660	-0.182	-0.113	-0.058	0.058	89.709
	$\theta = \tau = 0.01,\ h = 0.15$					$\theta = \tau = 0.05,\ h = 0.15$				
$\delta_{\theta,\tau}$	-0.049	-0.029	-0.016	0.014	95.049	-0.030	-0.018	-0.010	0.007	95.437
$\delta_{\theta,1/2}$	-0.041	-0.028	-0.019	0.011	99.223	-0.026	-0.018	-0.012	0.007	99.612
$\lambda_{\theta,\tau}$	-0.199	0.248	1.025	0.559	30.874	-0.081	0.212	0.902	0.471	40.485
$\lambda_{\theta,1/2}$	-0.163	0.214	0.788	0.387	41.650	-0.047	0.231	0.731	0.366	55.340
$100 \times \gamma_{\theta,\tau}$	-0.375	-0.194	-0.013	0.143	77.573	-0.242	-0.143	-0.061	0.076	88.932
$100 \times \gamma_{\theta,1/2}$	-0.300	-0.180	-0.045	0.099	86.214	-0.186	-0.115	-0.066	0.059	92.136
	$\theta = \tau = 0.01,\ h = 0.20$					$\theta = \tau = 0.05,\ h = 0.20$				
$\delta_{\theta,\tau}$	-0.048	-0.029	-0.017	0.013	97.087	-0.029	-0.018	-0.011	0.007	96.505
$\delta_{\theta,1/2}$	-0.041	-0.029	-0.020	0.011	99.515	-0.026	-0.018	-0.012	0.007	99.612
$\lambda_{\theta,\tau}$	-0.168	0.260	0.951	0.538	34.078	-0.056	0.227	0.890	0.475	44.078
$\lambda_{\theta,1/2}$	-0.093	0.154	0.624	0.308	47.184	-0.015	0.160	0.650	0.331	60.388
$100 \times \gamma_{\theta,\tau}$	-0.357	-0.197	-0.039	0.132	83.786	-0.231	-0.142	-0.068	0.071	90.583
$100 \times \gamma_{\theta,1/2}$	-0.299	-0.186	-0.059	0.092	88.252	-0.190	-0.118	-0.071	0.060	94.660

The table reports the summary statistics of the QL-CoVaR's parameters estimated for the N financial companies included in our dataset. We estimated the conditional quantiles for two quantile levels of θ and three bandwidth h levels. In each panel, from left to right, we report the following descriptive statistics of the coefficients: the 5-th percentile (5P), the median (MED), the 95-th percentile (95P), the interquartile range (IQR) and the percentage of times, out of N, in which they are statistically significant at the 5% confidence level (PS).

In contrast to the models analyzed above, the estimation process behind the QL-CoVaR (and thus the QL-CoCaViaR) depends on two additional parameters: a second quantile τ (we now restrict the attention on the neighbourhood of the τ-th quantile of $x_{i,t}$) and a bandwidth h (that calibrates the weight of the kernel function). Table 13 reports the statistics of the QL-CoVaR's coefficients, where we condition the estimates to the distress and to the median state of a single financial company. As before, the average impact exerted by the companies to both $QL\text{-}CoVaR_{\tau}^{(i)}$ and $QL\text{-}CoVaR_{1/2}^{(i)}$ is positive, but greater with respect to the standard CoVaR (the medians of both $\widehat{\lambda}_{\theta,\tau}^{(i)}$ and $\widehat{\lambda}_{\theta,0.5}^{(i)}$ are greater than the median of $\widehat{\lambda}_{\theta}^{(i)}$). Therefore, the relationships between the system and the companies become stronger by focusing on particular regions of the $x_{i,t}$ support, i.e. when $x_{i,t}$ is in a neighbourhood of a distress state.

Table 14: Estimation of $Q_{\theta,\tau}^{(i)}(y_t) = \delta_{\theta,\tau}^{(i)} + \psi_{1,\theta,\tau}^{(i)} Q_{\theta,\tau}^{(i)}(y_{t-1}) + \psi_{2,\theta,\tau}^{(i)}|y_{t-1}| + \lambda_{\theta,\tau}^{(i)} x_{i,t} + \gamma_{\theta,\tau}^{(i)} fpc_{t-1}$ and $Q_{\theta,1/2}^{(i)}(y_t) = \delta_{\theta,1/2}^{(i)} + \psi_{1,\theta,1/2}^{(i)} Q_{\theta,1/2}^{(i)}(y_{t-1}) + \psi_{2,\theta,1/2}^{(i)}|y_{t-1}| + \lambda_{\theta,1/2}^{(i)} x_{i,t} + \gamma_{\theta,1/2}^{(i)} fpc_{t-1}$.

COEF	5P	MED	95P	IQR	PS	5P	MED	95P	IQR	PS
	$\theta = \tau = 0.01$, $h = 0.10$					$\theta = \tau = 0.05$, $h = 0.10$				
$\delta_{\theta,\tau}$	-0.059	-0.020	-0.001	0.024	61.456	-0.033	-0.013	0.000	0.014	57.476
$\delta_{\theta,1/2}$	-0.041	-0.011	0.000	0.020	67.184	-0.026	-0.010	0.000	0.014	56.602
$\psi_{1,\theta,\tau}$	-0.698	0.181	0.919	0.829	45.534	-0.739	0.265	0.932	0.782	48.350
$\psi_{1,\theta,1/2}$	-0.582	0.491	0.957	0.842	65.049	-0.67	0.254	0.949	0.849	55.340
$\psi_{2,\theta,\tau}$	-0.602	-0.062	0.568	0.449	26.505	-0.355	-0.073	0.393	0.282	25.437
$\psi_{2,\theta,1/2}$	-0.570	-0.171	0.278	0.247	50.388	-0.318	-0.112	0.216	0.146	38.155
$\lambda_{\theta,\tau}$	-0.287	0.147	0.843	0.458	54.466	-0.125	0.121	0.855	0.437	53.01
$\lambda_{\theta,1/2}$	-0.250	0.086	0.906	0.379	51.942	-0.112	0.091	0.853	0.454	46.796
$100 \times \gamma_{\theta,\tau}$	-0.410	-0.113	0.004	0.173	52.913	-0.232	-0.089	0.001	0.117	52.718
$100 \times \gamma_{\theta,1/2}$	-0.233	-0.049	0.004	0.120	57.670	-0.177	-0.057	0.001	0.091	51.456
	$\theta = \tau = 0.01$, $h = 0.15$					$\theta = \tau = 0.05$, $h = 0.15$				
$\delta_{\theta,\tau}$	-0.054	-0.021	-0.001	0.023	67.184	-0.031	-0.013	0.000	0.013	61.068
$\delta_{\theta,1/2}$	-0.040	-0.012	0.00	0.021	73.592	-0.027	-0.011	0.000	0.014	65.146
$\psi_{1,\theta,\tau}$	-0.619	0.136	0.918	0.761	45.049	-0.746	0.221	0.927	0.754	47.670
$\psi_{1,\theta,1/2}$	-0.630	0.467	0.949	0.840	66.019	-0.662	0.220	0.937	0.830	55.437
$\psi_{2,\theta,\tau}$	-0.533	-0.060	0.479	0.386	27.184	-0.317	-0.085	0.323	0.237	29.029
$\psi_{2,\theta,1/2}$	-0.519	-0.172	0.234	0.219	54.660	-0.281	-0.116	0.156	0.129	41.748
$\lambda_{\theta,\tau}$	-0.218	0.174	0.902	0.496	57.573	-0.096	0.131	0.855	0.448	54.66
$\lambda_{\theta,1/2}$	-0.125	0.091	0.711	0.284	64.660	-0.047	0.109	0.722	0.396	57.573
$100 \times \gamma_{\theta,\tau}$	-0.402	-0.126	0.002	0.173	60.485	-0.220	-0.091	0.001	0.113	57.573
$100 \times \gamma_{\theta,1/2}$	-0.227	-0.053	0.003	0.128	63.592	-0.163	-0.068	0.000	0.089	60.777
	$\theta = \tau = 0.01$, $h = 0.20$					$\theta = \tau = 0.05$, $h = 0.20$				
$\delta_{\theta,\tau}$	-0.049	-0.022	-0.001	0.022	75.049	-0.03	-0.013	0.000	0.013	66.602
$\delta_{\theta,1/2}$	-0.042	-0.014	-0.001	0.022	80.971	-0.027	-0.012	0.000	0.014	72.913
$\psi_{1,\theta,\tau}$	-0.601	0.133	0.923	0.721	46.117	-0.687	0.176	0.921	0.745	46.699
$\psi_{1,\theta,1/2}$	-0.605	0.401	0.939	0.870	66.019	-0.660	0.186	0.927	0.823	55.437
$\psi_{2,\theta,\tau}$	-0.510	-0.074	0.384	0.333	32.816	-0.278	-0.087	0.283	0.204	32.330
$\psi_{2,\theta,1/2}$	-0.463	-0.175	0.198	0.205	55.437	-0.254	-0.122	0.117	0.121	42.816
$\lambda_{\theta,\tau}$	-0.194	0.185	0.886	0.525	64.757	-0.072	0.139	0.85	0.467	59.806
$\lambda_{\theta,1/2}$	-0.068	0.096	0.600	0.263	73.883	-0.022	0.103	0.652	0.351	65.243
$100 \times \gamma_{\theta,\tau}$	-0.343	-0.135	0.002	0.162	68.252	-0.207	-0.089	0.000	0.105	62.233
$100 \times \gamma_{\theta,1/2}$	-0.230	-0.067	0.001	0.138	70.874	-0.161	-0.071	-0.001	0.090	69.029

The table reports the summary statistics of the QL-CoCaViaR's parameters estimated for the N financial companies included in our dataset. We estimated the conditional quantiles for two quantile levels of θ and three bandwidth h levels. In each panel, from left to right, we report the following descriptive statistics of the coefficients: the 5-th percentile (5P), the median (MED), the 95-th percentile (95P), the interquartile range (IQR) and the percentage of times, out of N, in which they are statistically significant at the 5% confidence level (PS).

On average, we observe larger values for $\widehat{\lambda}_{\theta,\tau}^{(i)}$ at $\theta = 0.01$ than at $\theta = 0.05$, whereas the opposite holds for $\widehat{\lambda}_{\theta,0.5}^{(i)}$. $\widehat{\lambda}_{\theta,\tau}^{(i)}$ measures the relation between $x_{i,t}$ and y_t, when the companies and the system simultaneously lie in the left tail of their distributions. The fact that $\widehat{\lambda}_{\theta,\tau}^{(i)}$ increases as θ and τ simultaneously decrease means that the co-movements between the financial system and the companies become stronger when moving leftwards along the left tails of their distributions. Consequently, the risk of contagion increases by accentuating the distress degree in the connections between the financial system and the companies. The relevance of the co-movements between the financial system and the single companies emerges also in their median state. In fact, $\widehat{\lambda}^{(i)}_{\theta,0.5}$ increases as the system moves rightwards from $\theta = 0.01$ to $\theta = 0.05$, reducing the gap between the median state of the conditioning company and the distress state of the system.

Finally, the statistics of the QL-CoCaViaR's coefficients are given in Table 14. Similarly to the QL-CoVaR, on average, $\widehat{\lambda}^{(i)}_{\theta,\tau}$ takes higher values at $\theta = 0.01$ than $\theta = 0.05$ and the opposite holds for $\widehat{\lambda}^{(i)}_{\theta,0.5}$, highlighting the stronger co-movements between the system and the companies when they simultaneously lie in a state of accentuated distress. Switching from the QL-CoVaR to the QL-CoCaViaR, the inclusion of the CaViaR components absorbs, in part, the impact of the companies on the system: the medians of $\widehat{\lambda}^{(i)}_{\theta,\tau}$ and $\widehat{\lambda}^{(i)}_{\theta,0.5}$ are lower than the ones observed in the case of the QL-CoVaR. The effects of changing the h values are moderate in terms of the medians, the interquartile ranges, the 5-th and the 95-th percentiles of the coefficients; likewise, the changes in the h values do not imply relevant consequences in terms of times in which the coefficients are statistically significant over the N companies.

As in the case of the CoCaViaR, the values of $\psi^{(i)}_{1,\theta,\tau}$ and $\psi^{(i)}_{1,\theta,1/2}$ are positive on average, but lower than the ones typically observed in the standard CaViaR model. It is interesting to observe that the median of $\psi^{(i)}_{1,\theta,\tau}$ is lower at $\theta = \tau = 0.01$ than $\theta = \tau = 0.05$, in contrast to what occurs in the case of the CoCaViaR, where, as we said above, the average impact of $Q^{(i)}_{\theta,\tau}(y_{t-1})$ on $Q^{(i)}_{\theta,\tau}(y_t)$ is larger at $\theta = \tau = 0.01$. Differently, the median of $\psi^{(i)}_{1,\theta,0.5}$ is greater at $\theta = 0.01$ than $\theta = 0.05$. This phenomenon might be due to the fact that the persistence of the y_t quantiles over time is affected by the relations between the company and the system. In fact, as previously described, the co-movements between the system and the companies are stronger as θ and τ simultaneously take lower values and this absorbs, in part, the persistence of $Q^{(i)}_{\theta,\tau}(y_t)$. In contrast, when the system is in its distress state, whereas the i-th company is in its median state, the lower impact of their co-movements allows for a deeper persistence in the y_t quantiles over time. As a result, on average, $\psi^{(i)}_{1,0.01,0.01}$ is less than half $\psi^{(i)}_{1,0.01,0.5}$, whereas the differences between $\psi^{(i)}_{1,0.05,0.05}$ and $\psi^{(i)}_{1,0.05,0.5}$ are almost imperceptible. Similarly to the CoCaViaR, $|y_{t-1}|$ has a negative impact on $Q^{(i)}_{\theta,\tau}(y_t)$. $\psi^{(i)}_{2,\theta,\tau}$ is greater, in absolute value, at $\theta = \tau = 0.05$ than $\theta = \tau = 0.01$, whereas the opposite holds for $\psi^{(i)}_{2,\theta,0.5}$. Then, also the impact of the lagged system's returns on the current quantile of y_t is affected by the relationships between the system and the companies, according to the tranquil/distress period in which they stay. Furthermore, the changes in the h values have slight effects on the statistics of $\psi^{(i)}_{1,\theta,\tau}$, $\psi^{(i)}_{1,\theta,0.5}$, $\psi^{(i)}_{2,\theta,\tau}$ and $\psi^{(i)}_{2,\theta,0.5}$. Finally, contrasting results with respect to the case of the QL-CoVaR, we note that the number of companies for which the QL-CoCaViaR's coefficients are statistically significant at the 5% confidence level tends to be greater at the quantiles levels $(\theta, 0.5)$ than (θ, τ) and is almost always a positive function of h.

D Additional results

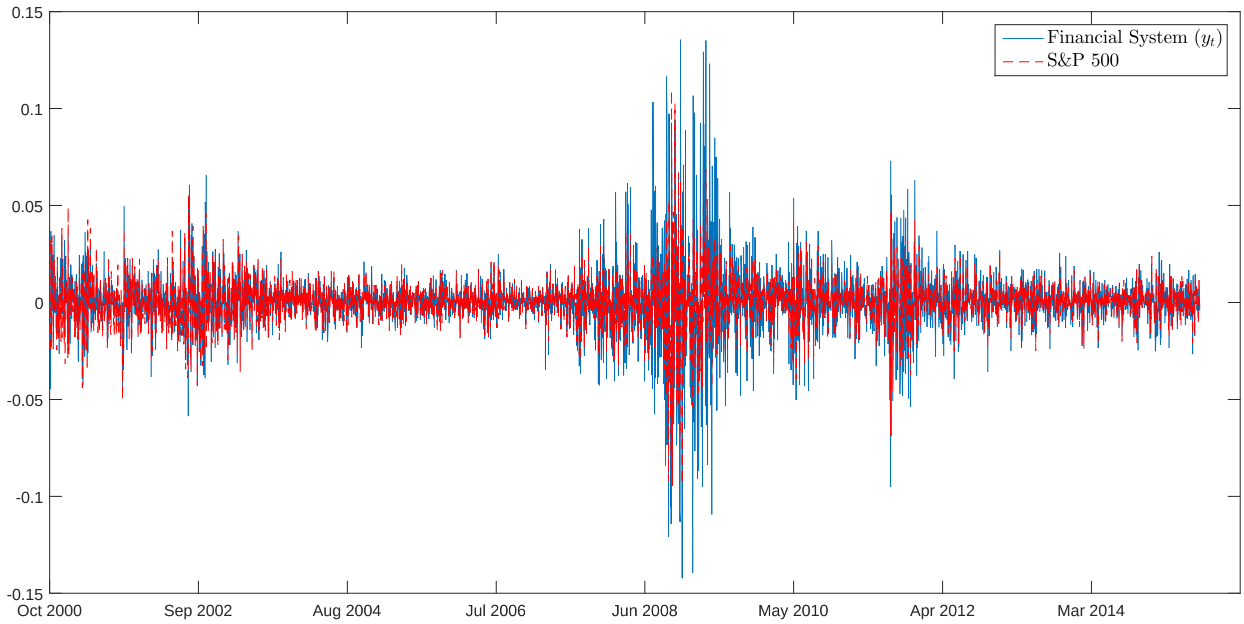

Figure 6: The figure displays the daily returns of the financial system (y_t) and of the Standard & Poor's 500 index (S&P 500) in the period October 10, 2000-July 31, 2015.

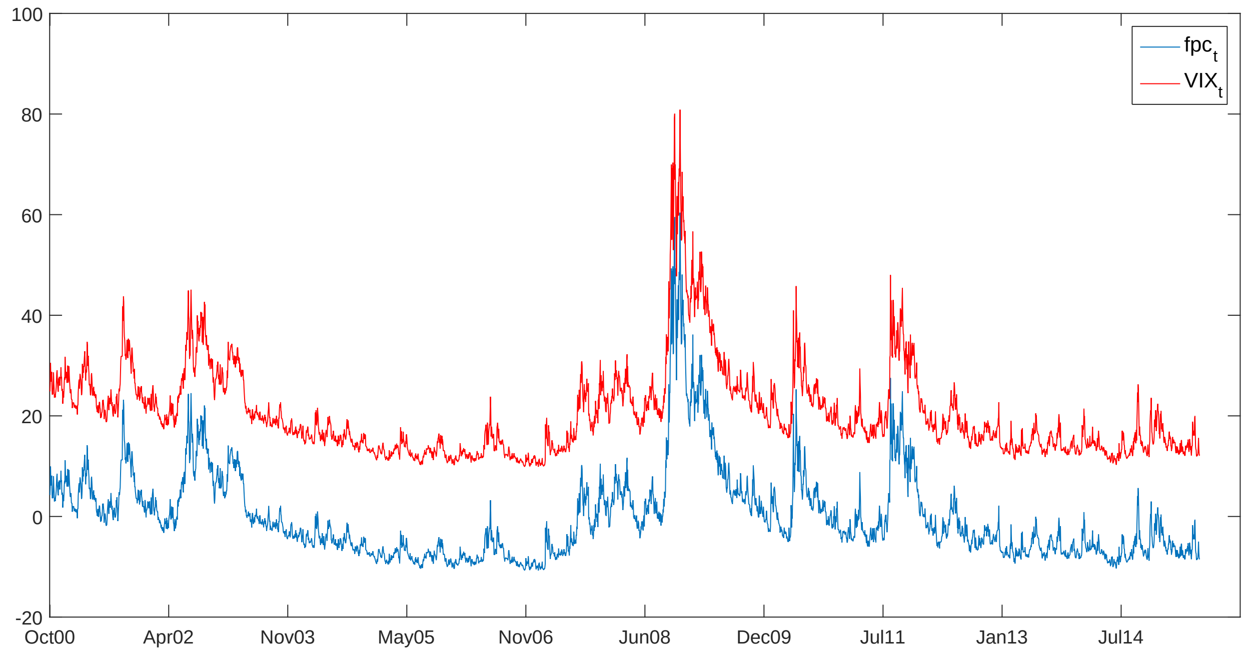

Figure 7: The figure compares the first principal component of the control variables included in $\mathbf{M}_t$ (fpc_t) and the VIX index (VIX_t) in the period October 10, 2000-July 31, 2015.

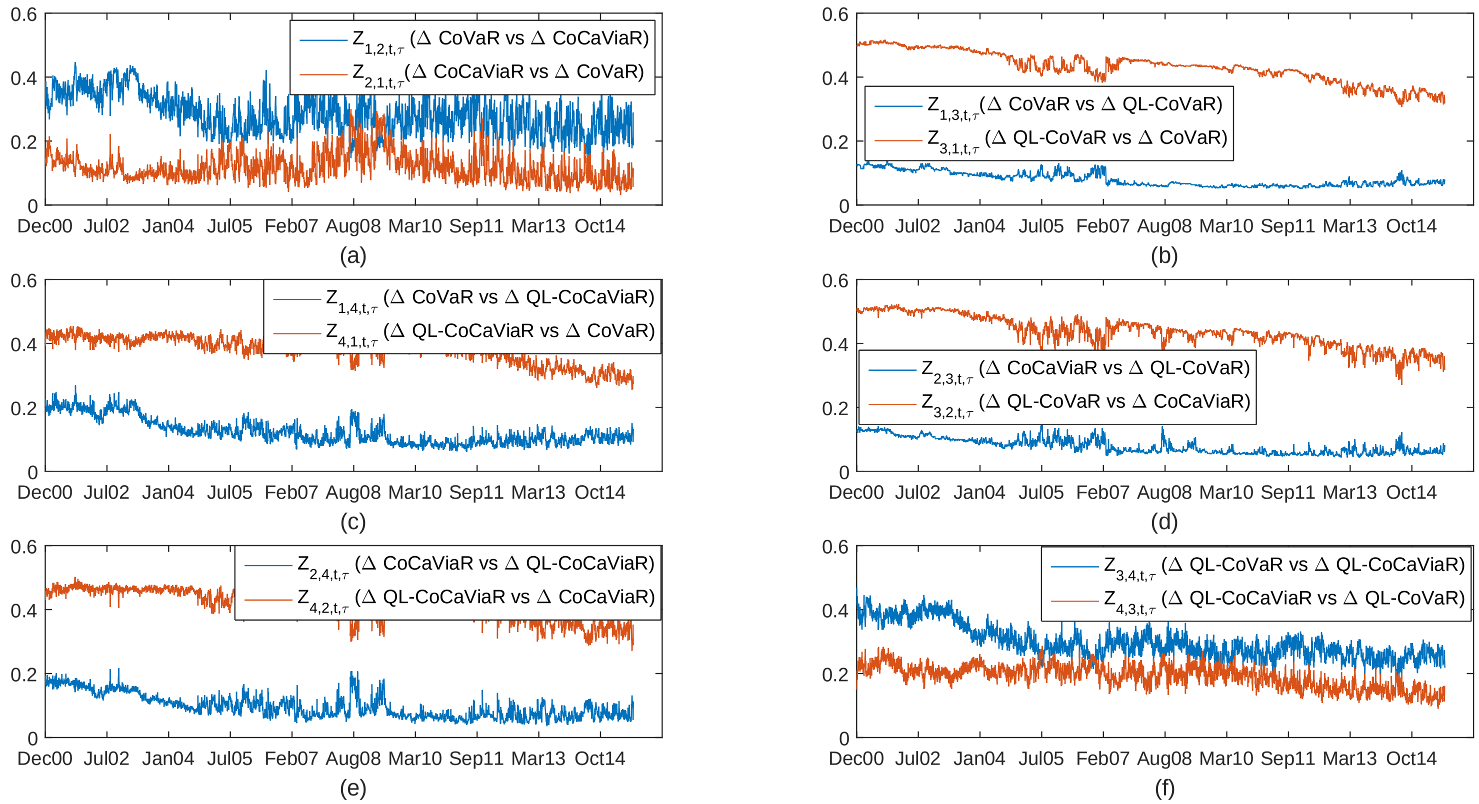

Figure 8: Trend over time of the indicator $Z_{j,k,t,\tau} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\left\{\Delta Risk^{(i)}_{j,t,\tau} - \Delta Risk^{(i)}_{k,t,\tau} < -\phi\right\}}$, where $\Delta Risk^{(i)}_{j,t,\tau}$ and $\Delta Risk^{(i)}_{k,t,\tau}$ are the j-th and the k-th elements of the set $S^{(i)}_{t,\tau} = \{100 \cdot \Delta CoVaR^{(i)}_{t,\tau}, 100 \cdot \Delta CoCaViaR^{(i)}_{t,\tau}, 100 \cdot \Delta QL\text{-}CoVaR^{(i)}_{t,\tau}, 100 \cdot \Delta QL\text{-}CoCaViaR^{(i)}_{t,\tau}\}$, for $j = 1, ..., 4$ and $k = 1, ..., 4$. $\mathbf{1}_{\{\cdot\}}$ is an indicator function taking value 1 if the condition in $\{\cdot\}$ is true, 0 otherwise. ϕ is equal to 10 basis points, whereas $\tau = 0.01$.

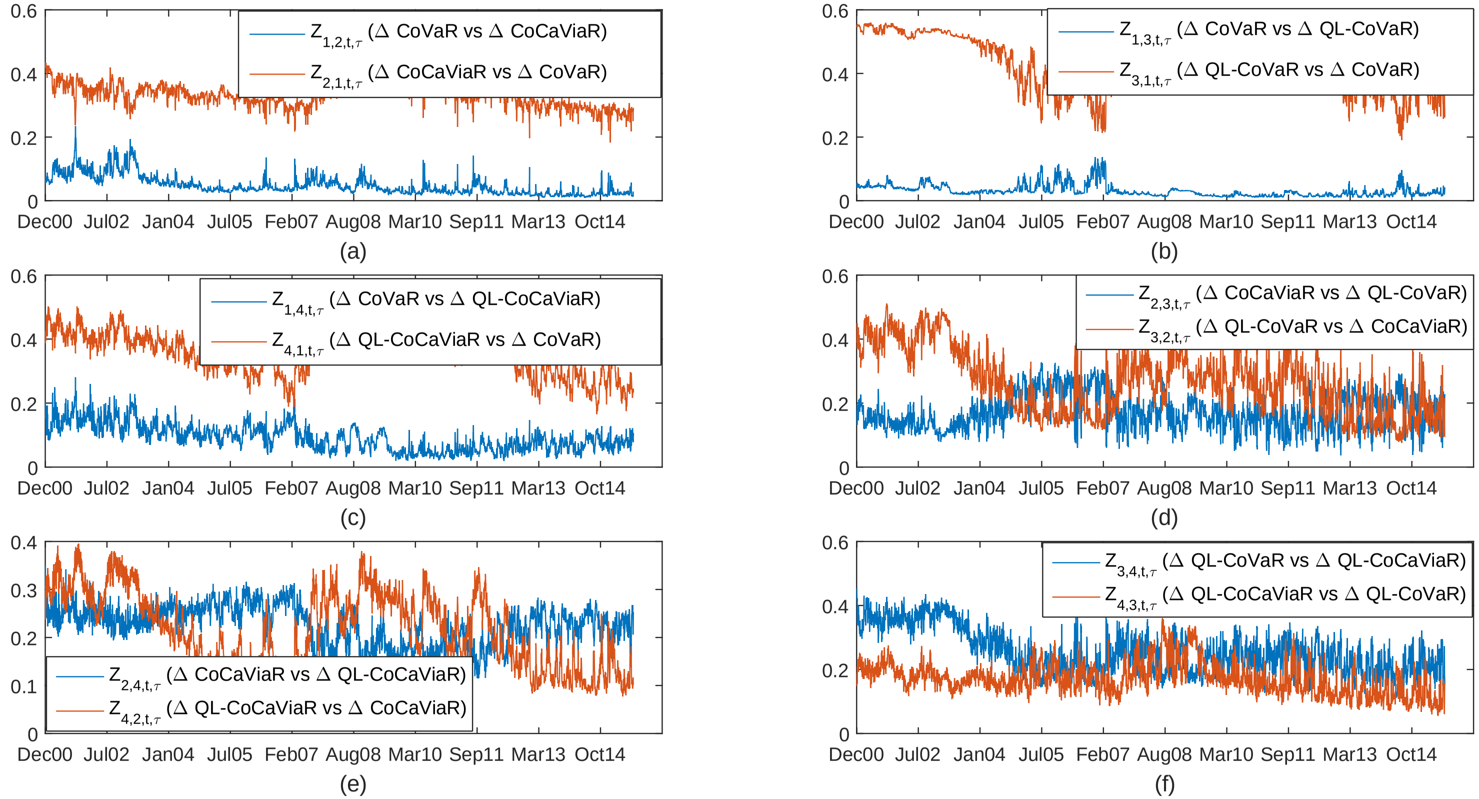

Figure 9: Trend over time of the indicator $Z_{j,k,t,\tau} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\left\{\Delta Risk_{j,t,\tau}^{(i)} - \Delta Risk_{k,t,\tau}^{(i)} < -\phi\right\}}$, where $\Delta Risk_{j,t,\tau}^{(i)}$ and $\Delta Risk_{k,t,\tau}^{(i)}$ are the j-th and the k-th elements of the set $S_{t,\tau}^{(i)} = \{100 \cdot \Delta CoVaR_{t,\tau}^{(i)}, 100 \cdot \Delta CoCaViaR_{t,\tau}^{(i)}, 100 \cdot \Delta QL\text{-}CoVaR_{t,\tau}^{(i)}, 100 \cdot \Delta QL\text{-}CoCaViaR_{t,\tau}^{(i)}\}$, for $j = 1, ..., 4$ and $k = 1, ..., 4$. $\mathbf{1}_{\{\cdot\}}$ is an indicator function taking value 1 if the condition in $\{\cdot\}$ is true, 0 otherwise. ϕ is equal to 10 basis points, whereas $\tau = 0.05$.

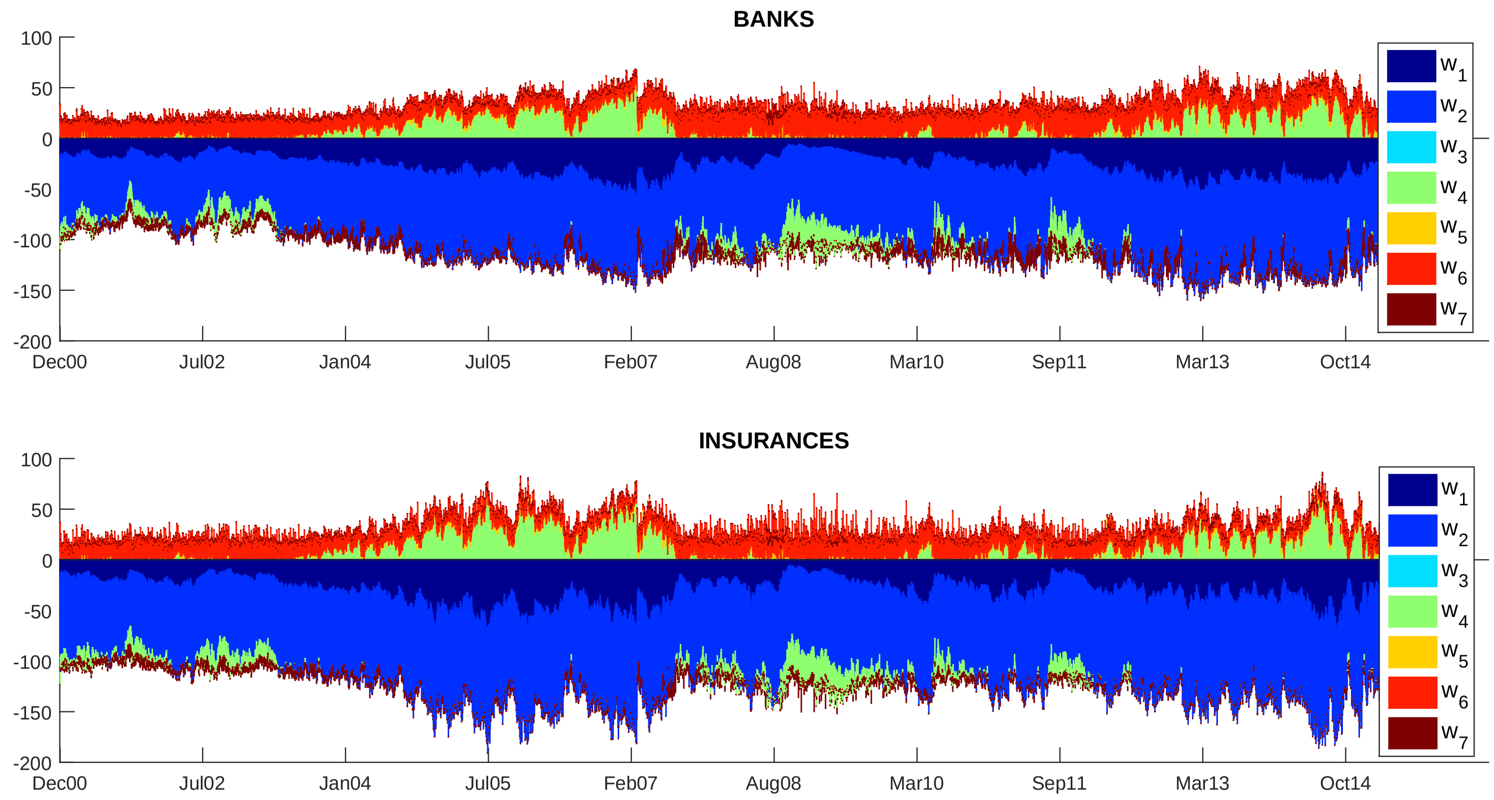

Figure 10: For each day from December 2000 to July 2015, the figure displays the cross-sectional medians of the $w_{i,\tau}$ ratios, for $i = 1, 2, \ldots 7$, distinguishing banks from insurances. The underlying risk measures are computed at $\theta = \tau = 0.01$ and $h = 0.15$.

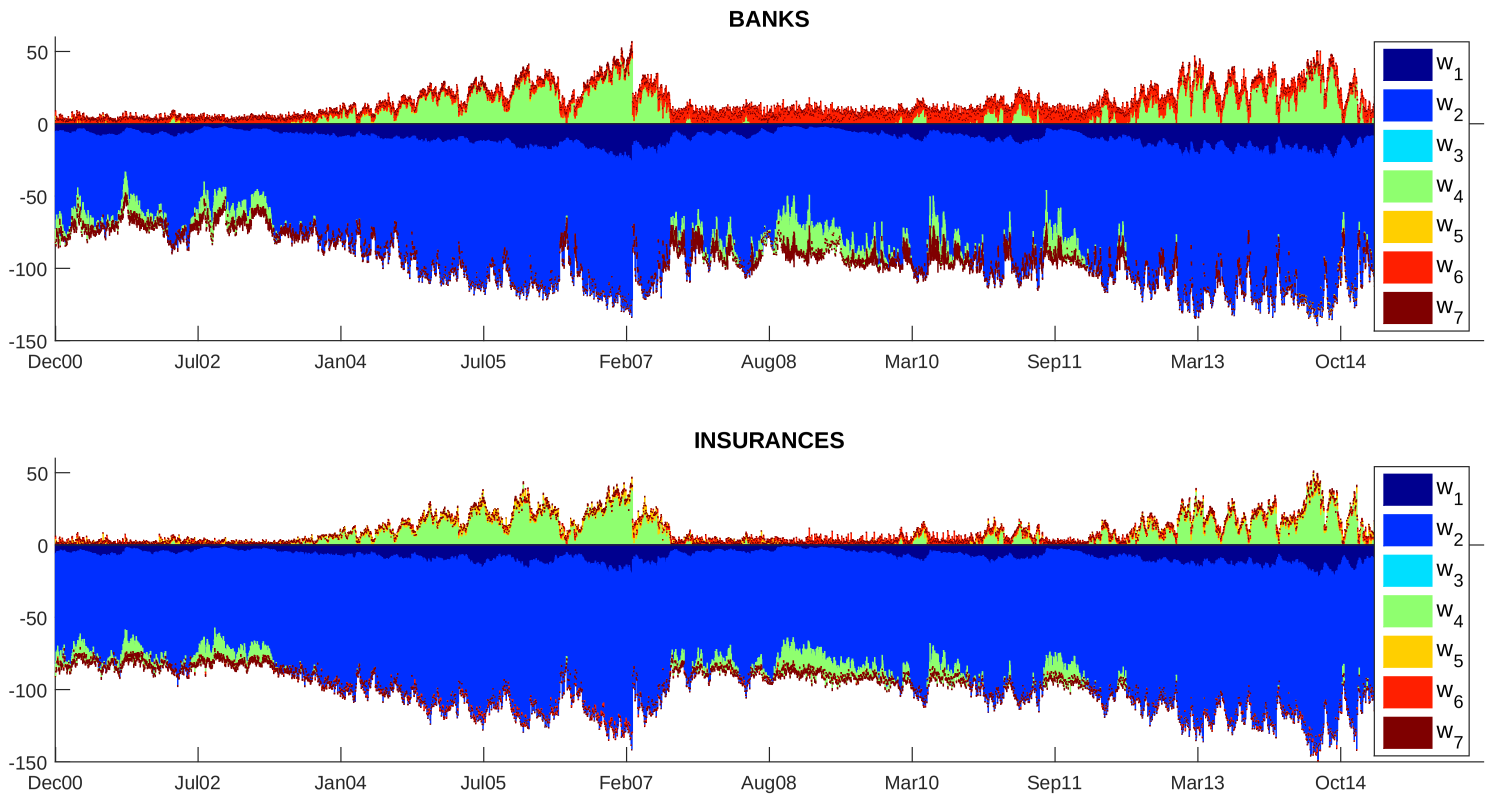

Figure 11: For each day from December 2000 to July 2015, the figure displays the cross-sectional medians of the $w_{i,\tau}$ ratios, for $i = 1, 2, \ldots 7$, distinguishing banks from insurances. The underlying risk measures are computed at $\theta = \tau = 0.05$ and $h = 0.15$.