Forecasting Optimal Portfolio Weights Using High Frequency Data

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Abstract

The paper evaluates the contribution of conditional second moments, from high frequency data, to optimal portfolio allocations. Using the DCC model as a benchmark, we put forth two novel approaches: a model for the inverse conditional correlation matrix (DCIC) and the direct modeling of the conditional portfolio weights (DCW). We assess their out-of-sample ability by comparing the corresponding minimum-variance portfolios built on the components of the Dow Jones 30 Index. Evaluating performance in terms of portfolio variance, certainty equivalent, turnover and break-even transaction costs, we find that exploiting conditional second moments gives marked improvements upon volatility timing and naïve strategies: DCC and the computationally convenient DCIC perform in a similar way; DCW, the simplest and fastest to implement, exhibits equal or superior performances with respect to the measures considered.

Keywords: Portfolio Allocation, Realized Volatility, Realized Correlations, Dynamic Conditional Modeling, Portfolio Weights Modeling.

JEL classification: C32, C53, G11, G17.

1 Introduction

Modeling and forecasting the temporal dependence in conditional second moments have key relevance in many areas of finance and in particular that of active portfolio management. Markowitz's (1952) optimal asset allocation, which requires forecasts of the first and second moments, has been extensively tested on low-frequency data with unsatisfying results. Focusing on expected returns, the use of in-sample (IS) estimates as forecasts usually leads to extreme weights which result in poor portfolio performances out-of-sample[1] (OOS). In order to contain forecast errors[2] and their negative effects on OOS portfolio performance, the literature has investigated a variety of approaches over the years: Bayesian diffuse priors, priors from asset pricing models, shrinkage, constraints from the factor structure of returns and the imposition of short-selling constraints, to mention a few. However, despite such efforts, the OOS evaluation of a representative number of these approaches[3] by De Miguel *et al.* (2009) has shown that none of them can consistently outperform the naïve equally-weighted portfolio: gains from optimal diversification are still offset by the forecast errors. Furthermore, in their simulation study calibrated to US data, De Miguel *et al.* (2009) show that all the approaches considered in their study, including Markowitz's mean-variance portfolio, require around 3,000 (6,000) observations for a portfolio with 25 (50) assets to outperform the naïve strategy.

Since the study of De Miguel *et al.* (2009) the literature has continued to investigate approaches that mitigate the negative impact of forecast errors. The shrinkage portfolio approach has been reconsidered and further developed by Behr *et al.* (2013) who include constraints, by De Miguel *et al.* (2013) who consider the shrinkage of the mean and the variance-covariance matrix and by De Miguel *et al.* (2014) who propose a multi-period shrinkage portfolio explicitly constructed around parameter uncertainty. Tu and Zhou (2011) advance the idea of shrinking the theoretically optimal portfolio weights toward the equal weights of the naïve strategy. Also focusing

[1]For a review relating theoretical portfolio choices to the data see Brandt (2007).

[2]In the literature, future expected returns are set equal to their sample averages over a given time period. In such setup reduction of the forecast errors corresponds to the reduction of the estimation error in average returns. However, since the ultimate goal is to reduce forecast errors, throughout the paper we directly refer to *forecast errors* instead of the more customary, but less general, *estimation error*.

[3]The Bayes-Stein shrinkage portfolio of Jorion (1985, 1986), the Bayesian portfolio based on belief in an asset-pricing model of Pástor (2000) and Pástor and Stambaugh (2000), the portfolio implied by asset-pricing models with unobservable factors of MacKinlay and Pástor (2000), the three-fund portfolio of Kan and Zhou (2007) and portfolio strategies with short-selling constraints.

on portfolio weights are Brandt *et al.* (2009) who propose to directly model each asset's portfolio weight as a function of its size, value and momentum characteristics. On the wake of this idea, Behr *et al.* (2012) suggest to consider industry- rather than firm-characteristics, to change the procedure with which the assets are ranked and to constrain portfolio weights to be non-negative (no short selling). Other approaches space from the inclusion of latent factors in addition to the factor-mimicking observables by Bouaddi and Taamouti (2013), to the use of option-implied volatility, risk premium and skewness to adjust expected returns by De Miguel *et al.* (2010) and the construction of low turnover portfolios based on volatility timing and reward-to-risk timing by Kirby and Ostdiek (2012).

Forecasting variance-covariance matrices, the other half of portfolio optimization, is a different story. Since the early works of Engle (1982) and Bollerslev (1986), it has been repeatedly shown that conditional variance models can produce accurate variance forecasts[4]. Improved forecasts may be obtained by modeling one of the different flavors of realized variance measures derived from high-frequency data[5].

Similarly, since the Dynamic Conditional Correlations (DCC) of Engle (2002b), correlation forecasts have also been shown to be quite accurate. Inherent to the modeling of positive definite conditional correlation matrices is the trade-off between parameters' parsimony and richness in the description of the second order dynamics. In fact, the number of parameters to be jointly estimated is a power function of the cross-sectional dimension M. For the DCC with *correlation targeting*[6] the order of parameters to be estimated in the full, diagonal and scalar versions are M^2, M^1 and M^0, respectively[7]. Analogously to the case of conditional variances, accurate multivariate realized measures of conditional covariances and correlations may be computed from high-frequency observations as in Barndorff-Nielsen *et al.* (2011) and directly modeled[8].

[4]For a review of forecasting with GARCH models see Andersen *et al.* (2006), Bauwens *et al.* (2012) and Teräsvirta (2012).

[5]Amongst the various approaches to volatility modeling that make use of realized measures are the Heterogeneous Autoregressive model (Har) of Corsi (2009) and Corsi *et al.* (2012), the Multiplicative Error Model (MEM) of Engle (2002a) and Brownlees *et al.* (2012) and the HEAVY of Shephard and Sheppard (2010) whose formulation is a particular case of the vector-MEM of Cipollini *et al.* (2013). For a survey see Andersen *et al.* (2003) and Park and Linton (2012), among others.

[6]In the original formulation of Engle and Mezrich (1996), *variance targeting* consists in setting a univariate volatility model's unconditional variance equal to its sample counterpart. Similarly, *correlation targeting* consists in setting a multivariate correlation model's unconditional correlation matrix equal to its sample counterpart, thus eliminating $M(M-1)/2$ parameters from the optimization procedure.

[7]For a review of Multivariate GARCH models see Bauwens *et al.* (2006).

[8]Numerous approaches to the modeling and forecasting of realized covariances and correlations

In this paper we focus on modeling and forecasting conditional second moments for optimal portfolio allocations. In particular, we consider the portfolio allocation problem of a *day trader* type of investor who closes positions at the end of each trading day. First of all, concentrating on day trading allows us to ignore overnight positions for which high frequency observations are not available; second, it allows us to neatly bypass all potential problems arising from short positions that stretch over long periods of time, as is the case in most of the previously cited studies . While the general setup of the paper is aimed at the mean-variance efficient portfolio, the empirical application exclusively concentrates on the minimum-variance portfolio allocation. As discussed in Section 2, focusing on the minimum-variance portfolio allows to isolate the contribution of conditional second moments forecasts to the optimal allocation problem. Besides proposing the modeling of the conditional inverse correlation matrix (DCIC), we also introduce the Dynamic Conditional Weights (DCW), a new multi-step modeling approach directed at optimal portfolio weights, that when associated with suitable estimation procedures, entirely circumvents the *curse of dimensionality* problem.

The paper is organized as follows. Section 2 introduces the optimal portfolio allocation problem. Models of the conditional second moments are presented in Section 3 while the direct modeling of the portfolio weights is introduced in Section 4. The objective functions considered for the estimation are discussed in Section 5. Data, measures of performance, and empirical results are described in Sections 6, 7 and 8, respectively. Section 9 concludes.

2 Optimal Portfolio Allocation

Let r_t be the $(M \times 1)$ vector of returns in excess of the risk-free rate. The conditional mean and variance-covariance matrix of r_t are denoted by μ_t and Ω_t, respectively. Investors, whose preferences are fully described by the portfolio's mean and variance, choose portfolio weights w_t to maximize expected quadratic utility:

$$V_t = \mathrm{w}_t'\mu_t - \frac{\gamma}{2}\mathrm{w}_t'\Omega_t\mathrm{w}_t$$

are either extensions of univariate realized variance models, adaptations of Multivariate GARCH models or both. Proposed modeling approaches are the fractionally integrated processes of Chiriac and Voev (2011), the vector autoregressions of Callot *et al.* (2017) and the specifications based on the Wishart distribution of Gourieroux *et al.* (2009), Golosnoy *et al.* (2012), Noureldin *et al.* (2012) and Jin and Maheu (2013), among others.

where γ is the investor's risk aversion. The optimal weights are $\mathrm{w}_t = \gamma^{-1}\Omega_t^{-1}\mu_t$ from which it may be seen that the level of risk aversion only determines the portfolio *leverage*, that is the fraction of wealth invested in the risk-free asset and the fraction to be invested across the risky assets. Consequently, the optimal *relative* portfolio weights ω_t, which define the optimal allocation within the risky assets, do not depend on risk aversion:

$$\omega_t = \frac{\Omega_t^{-1}\mu_t}{|\iota'\Omega_t^{-1}\mu_t|}$$

where ι is the $(M \times 1)$ unit vector. Notice that the weights ω_t are equally optimal for investors that either fix the portfolio expected return and choose ω_t to minimize the variance or fix the portfolio variance and choose ω_t to maximize the expected return. If expected returns may be treated as neither statistically nor economically different across assets, then the vector of expected returns may be expressed as $\mu_t = m_t \cdot \iota$, where m_t is a scalar. In turn, the vector of optimal *relative* portfolio weights becomes:

$$\omega_t = \frac{\Omega_t^{-1}\iota}{|\iota'\Omega_t^{-1}\iota|}$$

which corresponds to the allocation that minimizes the portfolio variance[9]. Similarly, when all wealth is invested in risky assets ($\mathrm{w}_t'\iota = 1 \Rightarrow \omega_t = \mathrm{w}_t$) and $\mu_t = m_t \cdot \iota$, the *absolute* portfolio weights that maximize expected quadratic utility correspond to the minimum-variance allocation. It must be emphasized that the minimum-variance portfolio has value beyond didacticism. First and foremost, bypassing the forecasts of expected returns allows to clearly evaluate the contribution of second moments modeling and forecasting to the optimal allocation. Second, empirically, the *minimum-variance* allocation has often been found to perform equally well as, if not better than, the *mean-variance*, even when measured in terms of Sharpe ratios[10].

3 Modeling Conditional Second Moments

Prior to the availability of high-frequency data and the development of the resulting realized measures, information about the conditional second moments had to be extracted, in general, from the outer product of the returns or their residuals after some filtration. This aspect has been crucial in forcing the various modeling approaches to the design of specifications for Ω_t. High-frequency data, instead, makes

[9]This allocation may also be seen as the limiting case of shrinkage estimators where all means are shrunk toward the common mean.

[10]See De Miguel *et al.* (2009), De Miguel *et al.* (2013) and De Miguel *et al.* (2014)

it possible to directly model Ω_t^{-1} provided that the realized measures are full rank[11]. Investigating the modeling possibilities of Ω_t^{-1} and their forecasting performances has value beyond that of an exercise in active portfolio management. Considering the case of the optimal mean-variance portfolio weights as a function of the *inverse* conditional variance-covariance matrix, two distinct modeling approaches emerge:

1. The standard decomposition of the conditional variance-covariance matrix in terms of standard-deviation D_t and correlation R_t matrices:

$$\Omega_t^{-1} = D_t^{-1} R_t^{-1} D_t^{-1} \tag{1}$$

2. The alternative decomposition of the *inverse* conditional variance-covariance matrix in terms of diagonal $\widetilde{D}_t$ and correlation $\widetilde{R}_t$ matrices:

$$\Omega_t^{-1} = \widetilde{D}_t \widetilde{R}_t \widetilde{D}_t \tag{2}$$

The decompositions in equations (1) and (2) are linked by the existence of the unique diagonal matrix F_t such that $\widetilde{D}_t = D_t^{-1} F_t$ and $\widetilde{R}_t = F_t^{-1} R_t^{-1} F_t^{-1}$. Specifically: D_t^{-1} of equation (1) collects the inverse conditional standard-deviations of the individual returns but R_t^{-1} is *not* a correlation matrix albeit being symmetric and positive definite; on the other hand, in equation (2), $\widetilde{D}_t$ collects the inverse conditional standard-deviations of linear combinations of all returns and $\widetilde{R}_t$ is a correlation matrix. In what follows we will focus on the standard decomposition of equation (1). Following the Multivariate GARCH literature, separation of conditional variances and correlations is maintained throughout the paper to allow for at least a two-step estimation procedure to ease the *curse of dimensionality* problem. D_t may be modeled element-by-element while the components of R_t may be modeled either jointly, as in DCC, or element-by-element in the order prescribed by the Sequential Conditional Correlations (SCC) decomposition of Palandri (2009). The use of high-frequency data and realized measures allows to expand these pre-existing possibilities to the direct modeling[12] of R_t^{-1}. From a computational perspective, this is more convenient than modeling R_t whenever the objective function of the estimator requires calculations of the inverse, as in Section 5.2.

[11] Although realized measures that are not full rank are not immediately invertible, there are expedients that may be used to circumvent this deficiency such as pre-averaging.

[12] Abandoning invertibility and positive definiteness, R_t^{-1} may be modeled and estimated element-by-element without necessarily hindering its usefulness as is the case for R_t. We leave these modeling possibilities at a declaration stage and focus instead on specifications of R_t^{-1} that preserve invertibility and positive definiteness.

3.1 Variance Modeling

Let $\widehat{\sigma}^2_{i,t}$ be a realized measure of the variance of asset i at time t and $\sigma^2_{i,t} \equiv \mathbb{E}_{t-1}[\widehat{\sigma}^2_{i,t}]$ its conditional expectation at $(t-1)$. For comparisons purposes we consider the following dynamic (1,1) specifications[13]:

1. The Garch parameterization of $\sigma^2_{i,t}$:

$$\sigma^2_{i,t} = c_i + \alpha_i \cdot \widehat{\sigma}^2_{i,t-1} + \beta_i \cdot \sigma^2_{i,t-1}$$

2. The LnGarch which parameterizes the log-conditional variance:

$$\ln(\sigma^2_{i,t}) = c_i + \alpha_i \cdot \ln\left(\widehat{\sigma}^2_{i,t-1}\right) + \beta_i \cdot \ln(\sigma^2_{i,t-1})$$

3. The InvGarch parameterization of the conditional precision $\sigma^{-2}_{i,t}$, the quantity entering the optimal portfolio weights equation:

$$\sigma^{-2}_{i,t} = c_i + \alpha_i \cdot \widehat{\sigma}^{-2}_{i,t-1} + \beta_i \cdot \sigma^{-2}_{i,t-1}$$

4. As a benchmark to the above specifications we consider the popular Har of Corsi (2009) which models the conditional variance $\sigma^2_{i,t}$ as function of past realizations over daily, weekly and monthly time intervals:

$$\sigma^2_{i,t} = c_i + \alpha_{i,1} \cdot \widehat{\sigma}^2_{i,t-1} + \alpha_{i,2} \cdot \frac{1}{5}\sum_{j=1}^{5} \widehat{\sigma}^2_{i,t-j} + \alpha_{i,3} \cdot \frac{1}{22}\sum_{j=1}^{22} \widehat{\sigma}^2_{i,t-j}$$

3.2 Dynamic Conditional Correlations Modeling

Let $\widehat{R}_t$ be a realized measure of the correlation matrix of M assets at time t and $R_t \equiv \mathbb{E}_{t-1}[\widehat{R}_t]$ its conditional expectation at $(t-1)$. The DCC(1,1) parameterization with *targeting* models R_t according to:

$$R_t = \left(\overline{R} - A\overline{R}A' - B\overline{R}B'\right) + A\widehat{R}_{t-1}A' + BR_{t-1}B' \tag{3}$$

where $\overline{R}$ is the sample unconditional correlation matrix and A and B are either full, diagonal or scalar matrices of parameters. For scalar and diagonal matrices of coefficients, R_t on the left-hand-side of equation (3) is guaranteed to be a correlation matrix. The same is not true when the matrices of coefficients are full. In this case, the left-hand-side quantity of equation (3) is standardized by the elements on its main diagonal to produce a correlation matrix.

[13]These are well suited in the empirical applications as highlighted by Hansen and Lunde (2005); in general, further lags could be considered.

3.3 Dynamic Conditional Inverse Correlations Modeling

An alternative to DCC that circumvents the trade-off between parameter parsimony and richness in the description of the second order dynamics is the SCC which decomposes R_t into correlations and partial correlations. This eliminates the *curse of dimensionality* by allowing for multi-step estimation of such elements and straightforward reconstruction of positive definite correlation matrices. Since applying the SCC methodology to realized measures of the conditional correlation matrix is straightforward, rather than re-considering that methodology we propose two new approaches. To begin with, we propose the Dynamic Conditional Inverse Correlation DCIC (1,1) parameterization with *targeting*, derived from the observation that using realized measures it is possible to model R_t^{-1} directly:

$$R_t^{-1} = \left(\overline{R}^{-1} - A\overline{R}^{-1}A' - B\overline{R}^{-1}B'\right) + A\widehat{R}_{t-1}^{-1}A' + BR_{t-1}^{-1}B' \tag{4}$$

While a linear combination of correlation matrices is a correlation matrix itself, the same is not true for their inverses[14]. Although this aspect is of no relevance for the estimation procedure, proper forecasts of inverse correlation matrices may be obtained following the same standardization procedure of DCC. Specifically, the legitimate inverse correlation matrix $\mathring{R}_t^{-1}$ is given by:

$$\mathring{R}_t^{-1} = \mathring{D}_t \, R_t^{-1} \, \mathring{D}_t$$

where $\mathring{D}_t$ is a diagonal matrix with elements equal to the square-root of the diagonal elements of $\left[R_t^{-1}\right]^{-1}$.

4 Dynamic Conditional Weights Modeling

An advantageous direct parameterization of the conditional portfolio weights is that of the proposed Dynamic Conditional Weights DCW. It is easier to implement than SCC, as it does not require to follow SCC's specific decomposition of the conditional correlation matrix, and is less computationally demanding than the approaches to conditional correlations, as it only requires the modeling of a number of dynamic components equal to the cross-section of the data. Therefore, direct modeling of the weights is no more complicated than the estimation of a factor model within the Arbitrage Pricing Theory framework.

[14] If X and Y are correlation matrices and λ a scalar, it follows that $\lambda X + (1-\lambda)Y$ is a correlation matrix while, in general, $(\lambda X^{-1} + (1-\lambda)Y^{-1})^{-1}$ is not a correlation matrix.

The proposed direct modeling of the optimal portfolio weights may be seen as an extension to a different setting of the works of Brandt *et al.* (2009) and Engle and Manganelli (2004). The former introduce a static model for the portfolio weights where the explanatory variables are among those suggested by the asset-pricing literature and which at best are predictive for the conditional expected returns component but are unlikely candidates for the conditional expected variance-covariance matrix component. On the other hand, the latter, introduce a dynamic model for the conditional Value at Risk which condenses into one equation the dynamics of the time-varying conditional variances and the distribution quantile of interest.

Let $\widehat{\mathrm{w}}_t$ be the $(M \times 1)$ vector of realized optimal portfolio weights at time t and $\mathrm{w}_t \equiv \mathbb{E}_{t-1}[\widehat{\mathrm{w}}_t]$ its conditional expectation at $(t-1)$. Consistently with the corresponding literature, the conditional expected returns μ_t may be treated as *slow-moving* relative to daily frequencies. Therefore, the return forecasts $\widehat{\mu}_t$ may be set equal to μ for every t over some time interval $\mathcal{T}$ coherent with long-horizon forecastability. As a result:

$$\mathrm{w}_t = \Omega_t^{-1}\mu \quad \text{and} \quad \widehat{\mathrm{w}}_t = \widehat{\Omega}_t^{-1}\mu$$

Assuming that the elements of Ω_t^{-1} follow a common GARCH process yields:

$$\begin{aligned} \mathrm{w}_t &= \left[\Omega^{-1} + \alpha\widehat{\Omega}_{t-1}^{-1} + \beta\Omega_{t-1}^{-1}\right]\mu \\ &= \kappa + \alpha\widehat{\mathrm{w}}_{t-1} + \beta\mathrm{w}_{t-1} \end{aligned} \tag{5}$$

where $\widehat{\Omega}_{t-1}^{-1}$ is the inverse of the realized variance-covariance matrix. It is straightforward to generalize the dynamic specification of equation (5) to *relative* portfolio weights, higher order lags and *non-scalar* matrices of coefficients. In particular, let $\widehat{\omega}_t$ be the $(M \times 1)$ vector of *relative* realized optimal portfolio weights at time t and $\omega_t \equiv \mathbb{E}_{t-1}[\widehat{\omega}_t]$ its conditional expectation at $(t-1)$. Then, the (1,1) parameterization with *targeting* is:

$$\omega_t = (I - A - B)\,\overline{\omega} + A\widehat{\omega}_{t-1} + B\omega_{t-1} \tag{6}$$

where $\overline{\omega}$ is the sample average of the optimal weights and A and B are either full, diagonal or scalar matrices of parameters. When the matrices A and B are scalar, the elements of ω_t from equation (6) will add to unity by construction as long as $\iota'\omega_0 = 1$. In all other cases, $\iota'\omega_t \neq 1$ and the standardization of the weights $\omega_t/(\iota'\omega_t)$ is needed in both phases of estimation and forecasting.

5 Objective Functions

The parameters of the models of sections 3 and 4 may be estimated, among others, by Least Squares (LS) or Quasi Maximum Likelihood (QML). While QML estimation may be appealing for its theoretical properties[15], the LS estimator is particularly attractive for the ease and speed with which it delivers the parameters' point estimates. Next to these, for the DCW only, we explore the possibility of using the portfolio variance (OP) itself as the estimator's objective function.

5.1 Least Squares (LS)

The LS objective function simply measures the distance between predictions and realizations. For the estimation of conditional variance models:

$$\sum_{t=1}^{T} \left[f(\sigma^2_{i,t}) - f(\widehat{\sigma}^2_{i,t}) \right]^2 \qquad \forall i = 1, \ldots, M$$

where f is the identity function for Garch and Har, the inverse function for InvGarch and the logarithmic function for LnGarch. Similarly, for the estimation of the conditional correlation models:

$$\sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{j=i}^{M} \left[R_{i,j,t} - \widehat{R}_{i,j,t} \right]^2 \tag{7}$$

Notice that, for the DCC specifications, the elements on the main diagonal are equal to unity both in R_t and $\widehat{R}_t$ and therefore cancel out. On the other hand, should equation (7) be used to estimate DCIC specifications, elements on the main diagonal would not cancel out. Specifically, the distinct $M(M-1)/2$ correlations of R_t are *non-linearly* mapped into the $M(M+1)/2$ on- and off-diagonal elements of R_t^{-1}. As M grows, the LS objective function of equation (7) gives relatively less weight to the diagonal elements and more weight to those off-diagonal. Precisely, the weight of the elements on the main diagonal, relative to those off-diagonal, is $2/(M-1)$. Therefore, unless the dynamic properties of all the elements of R_t^{-1} are the same, for large M, the LS objective function specified in equation (7) will squander the information content of the elements on the main diagonal. This aspect clearly suggests the need for a different quadratic distance, agreeably one specially designed for inverse correlation

[15] Quadratic-exponential density functions guarantee consistency of the parameters' estimates and when the *true* density is *similar* to the chosen quasi likelihood function the efficiency of the QML estimator may be expected to be *similar* to that of Maximum-Likelihood.

matrices. With no presumption of exhaustively addressing this delicate issue here, we introduce the following simple rebalancing of the elements of the LS objective function for DCIC specifications:

$$\sum_{t=1}^{T} \left\{ \frac{(M-1)}{2} \sum_{i=1}^{M} \left[R_{i,i,t}^{-1} - \widehat{R}_{i,i,t}^{-1} \right]^2 + \sum_{i=1}^{M} \sum_{j=i+1}^{M} \left[R_{i,j,t}^{-1} - \widehat{R}_{i,j,t}^{-1} \right]^2 \right\} \tag{8}$$

The LS estimation of the DCW model of equation (6) may be easily performed from the objective function:

$$\sum_{t=1}^{T} \sum_{i=1}^{M} \left[\omega_{i,t} - \widehat{\omega}_{i,t} \right]^2$$

If and only if the DCW matrices of coefficients are diagonal, the LS estimation simplifies to an extremely convenient element-by-element modeling and estimation with objective functions:

$$\sum_{t=1}^{T} \left[\omega_{i,t} - \widehat{\omega}_{i,t} \right]^2 \qquad \forall i = 1, \ldots, M$$

5.2 Quasi Maximum Likelihood (QML)

The concentrated Gaussian log-likelihood function for the estimation of the conditional variance models of Section 3.1 is given by:

$$l = -\sum_{t=1}^{T} \left[\ln \sigma_{i,t}^2 + \frac{\widehat{\sigma}_{i,t}^2}{\sigma_{i,t}^2} \right] \qquad \forall i = 1, \ldots, M$$

where $\sigma_{i,t}^2$ and $\widehat{\sigma}_{i,t}^2$ are the conditional- and the realized-variance, respectively. Similarly, the concentrated Gaussian log-likelihood function[16] for the estimation of the conditional models of Sections 3.2 and 3.3 is given by:

$$l = -\sum_{t=1}^{T} \left[\ln |R_t| + \mathbb{TR} \left(R_t^{-1} \widehat{R}_t \right) \right] \tag{9}$$

where $\mathbb{TR}$ is the trace and R_t and $\widehat{R}_t$ are the conditional and the realized correlation matrices, respectively. For non-trivial cross sectional dimensions M, the QML estimation of DCC specifications is hindered by the computationally intensive calculations of the determinant and the inverse of R_t at every t and for every iteration of the optimizer. On the other hand, QML estimation of DCIC specifications are substantially

[16] The same Gaussian likelihood is used by Noureldin *et al.* (2012) for the estimation of multivariate HEAVY models.

less intensive as they do not require matrix inversions: R_t^{-1} is readily available and $\ln|R_t| = -\ln|R_t^{-1}|$.

For comparative purposes, the DCW specification of Section 4 may also be reconducted to a QML estimation. In particular, since the portfolio weights $\omega_{i,j}$ and $\widehat{\omega}_{i,j}$ span $(-\infty, +\infty)$, their Fisher transformations[17] $\chi_{i,t}$ and $\widehat{\chi}_{i,t}$ span $(-1, +1)$. Therefore, treating $\chi_{i,t}$ and $\widehat{\chi}_{i,t}$ as conditional and realized correlations, respectively, QML estimations of the model parameters may be performed using the bivariate specification of the Gaussian log-likelihood function of equation (9):

$$l = -\sum_{t=1}^{T}\left[\ln(1-\chi_{i,t}^2) + 2\cdot\frac{1-\chi_{i,t}\widehat{\chi}_{i,t}}{1-\chi_{i,t}^2}\right] \qquad \forall i = 1,\ldots,M$$

Constructions of the Gaussian log-likelihood functions may be found in Appendix A.2.

5.3 Portfolio Optimization (OP)

The parameters of the minimum-variance DCW specification may be estimated by minimizing the overall portfolio variance (OP):

$$\sum_{t=1}^{T}\omega_t'\widehat{\Omega}_t\omega_t \tag{10}$$

On the one hand, this objective function[18] has the appealing property of *training* the weights specifications to optimize given properties of the portfolio that are of primary interest. On the other hand, it has the same drawback of the QML approach as it does not avoid jointly estimating the model parameters. In fact, for DCC and DCIC, the objective function (10) would require the impractical joint estimation of all the conditional variance and conditional correlation models. For this reason, feasibility and goodness of the objective function of equation (10) are evaluated in the empirical analysis only for DCW.

6 Data

The data used for portfolio selection pertains to $M = 28$ of the 30 constituents of the Dow Jones 30 Index. The sample has 12 years of high-frequency daily observations

[17] The Fisher transformation of $\omega_{i,t}$ is $\chi_{i,t} = (e^{2\omega_{i,t}}-1)\cdot(e^{2\omega_{i,t}}+1)^{-1}$ from which *mutatis mutandis* follows that of $\widehat{\omega}_{i,t}$.

[18] Similarly, for the mean-variance portfolio, the DCW specification may be estimated by maximizing the quadratic utility: $\sum_{t=1}^{T}\left[\mathrm{w}_t'\widehat{\mu}_t - \frac{\gamma}{2}\mathrm{w}_t'\widehat{\Omega}_t\mathrm{w}_t\right]$.

from 01/03/2005 to 12/31/2015 for a total of 2768 days. The two series, with tickers TRV and V, are not included in the study because they are not available for the full sample period[19]. Tickers of the 28 included stocks are: AAPL, AXP , BA, CAT, CSCO, CVX, DD, DIS, GE, GS, HD, IBM, INTC, JNJ, JPM, KO, MCD, MMM, MRK, MSFT, NKE, PFE, PG, UNH, UTX, VZ, WMT, XOM. The raw tick-by-tick TAQ data is cleaned using the procedure of Brownlees and Gallo (2006) from which realized kernel covariances are computed following the approach of Barndorff-Nielsen *et al.* (2011). Details on this procedure may be found in Appendix A.1.

The sample is split into six 5-year IS periods: 2005-2009 (1259 obs.), 2006-2010 (1259 obs.), 2007-2011 (1260 obs.), 2008-2012 (1259 obs.), 2009-2013 (1258 obs.) and 2010-2014 (1258 obs.). All model combinations are estimated on each of the six IS periods, and for each of them, OOS forecasts are generated for the following 1-year period: 2010 (252 obs.), 2011 (252 obs.), 2012 (250 obs.), 2013 (252 obs.), 2014 (252 obs.) and 2015 (251 obs.).

The large cap characteristic and the number of stocks considered are in line with those of the assets examined in De Miguel *et al.* (2009). Specifically, the assets they use to construct the optimal portfolios are portfolios themselves and therefore primarily large caps: 10 sector portfolios, 10 industry portfolios, 8 country indices and 20 size and book-to-market portfolios treated as separate data-sets. Similarly, the various cross-sections of assets in their study range between $M = 3$ and $M = 24$. In the design of our study, $M = 28$ allows for the comparison of DCW with standard parameterizations that suffer from the *curse of dimensionality* whenever flexible dynamics of the various components are needed. Furthermore, since the correlations of large caps are expected to co-move more than those between large and small caps, the chosen data-set is, at least in theory, tailor-made for the Scalar DCC making it a challenging benchmark for competing models.

[19]TRV data are available only from 02/26/2007 while V data are missing from 08/04/2006 to 02/26/2007.

7 Measures of Portfolio Performance

7.1 Portfolio Variance (PV)

One measure of OOS performance is the average portfolio variance[20] that emerges from choosing model or model-combination κ:

$$\mathsf{PV}_{\kappa} = \frac{1}{T}\sum_{t=1}^{T} \omega_{\kappa,t}\widehat{\Omega}_t\omega_{\kappa,t}$$

where $t = 1, \ldots, T$ is the OOS period, $\omega_{\kappa,t}$ are the forecasts of the optimal portfolio weights from model or model-combination κ and $\widehat{\Omega}_t$ is the OOS realized variance-covariance matrix.

7.2 Certainty Equivalent Return (CEQ)

Another common measure of OOS performance is the certainty equivalent return. Defined as the certain return that an investor is willing to accept in order to abandon a risky strategy, the certainty equivalent return highlights reward-to-risk too, although in a different manner. Similarly, the certainty equivalent return may be defined as the certain return that an investor is willing to accept to switch from model or model-combination κ_1 to κ_2:

$$\mathsf{CEQ}_{\kappa_1\to\kappa_2} = \frac{1}{T}\sum_{t=1}^{T}\left[\omega'_{\kappa_1,t}\widehat{\mu}_t - \frac{\gamma}{2}\omega'_{\kappa_1,t}\widehat{\Omega}_t\omega_{\kappa_1,t} - \omega'_{\kappa_2,t}\widehat{\mu}_t + \frac{\gamma}{2}\omega'_{\kappa_2,t}\widehat{\Omega}_t\omega_{\kappa_2,t}\right] \tag{11}$$

If the certainty equivalent return is positive (negative), the investor requires an average payment of (is willing to pay) $\mathsf{CEQ}_{\kappa_1\to\kappa_2}$ to switch from κ_1 to κ_2. Coherently with the aim of this study, an OOS measure that only captures second moment effects may be obtained either by ignoring the OOS excess returns, or by imposing that they are the same across specifications: $\omega'_{\kappa,t}\widehat{\mu}_t = \tilde{\mu}_t$ for every κ. Therefore, equation (11) simplifies to:

$$\mathsf{CEQ}_{\kappa_1\to\kappa_2} = \gamma \cdot \frac{1}{2}\left(\mathsf{PV}_{\kappa_2} - \mathsf{PV}_{\kappa_1}\right) \tag{12}$$

With this formulation, reporting $\mathsf{CEQ}_{\kappa_1\to\kappa_2}$ for $\gamma = 1$ allows for the immediate calculation of the certainty equivalent return for any value of risk aversion[21] simply by rescaling the reported value by γ.

[20] A common measure of the OOS portfolio performance is the Sharpe ratio which highlights the reward-to-risk. However, given that in this study we concentrate exclusively on the contribution of the conditional second moments to optimal portfolio formation, we deem it more appropriate to use a measure of OOS performance that captures second moment effects only.

[21] For example, De Miguel *el al.* (2009) consider risk aversion coefficients of $\gamma = \{1, 2, 3, 4, 5, 10\}$.

7.3 Turnover (TO)

In this study, where the focus is on daily trading with no overnight holdings, we have zero portfolio weights prior to rebalancing. Hence, average turnover is given by:

$$\mathsf{TO}_\kappa = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{M}|\omega_{\kappa,j,t}|$$

which is a measure of the average portfolio *leverage* $\overline{LV}_\kappa$[22] of the forecasting model κ.

7.4 Break-Even Transaction Costs (BETC)

Let us first introduce the number of shares $n_{\kappa,j,t}$ of asset j purchased or sold at the beginning of trading day t in accordance to the forecasts of model κ: $n_{\kappa,j,t} = |\omega_{\kappa,j,t}|/P_{j,t}^{o}$, given the opening price of the asset $P_{j,t}^{o}$. Assuming markup transaction costs τ[23], it follows that the associated cost is $\tau n_{\kappa,j,t}P_{j,t}^{o} = \tau|\omega_{\kappa,j,t}|$. Similarly, the cost arising from closing the position at the end of the day is $\tau n_{\kappa,j,t}P_{j,t}^{c} = \tau(1+R_{j,t}^{oc})|\omega_{\kappa,j,t}|$, where $P_{j,t}^{c}$ and $R_{j,t}^{oc}$ are the closing price and the open-to-close return in t of asset j, respectively. Summing open and close costs for each asset j gives the transaction costs of holding the portfolio of model κ. Averaging over T, we get the average transaction costs of the forecasting model κ:

$$\overline{TC}_\kappa = \tau\cdot\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{M}(2+R_{j,t}^{oc})\cdot|\omega_{\kappa,j,t}| \tag{13}$$

As shown in Appendix A.3, $\overline{TC}_\kappa$ may be approximated up to two orders of magnitude by TC_κ:

$$TC_\kappa \approx 2\tau\cdot\mathsf{TO}_\kappa$$

Combining the above with equation (12) allows to derive the *net* certainty equivalent return:

$$NCEQ_{\kappa_1\to\kappa_2} = \gamma\cdot\frac{1}{2}\left(\mathsf{PV}_{\kappa_2}-\mathsf{PV}_{\kappa_1}\right) + 2\tau\left(\mathsf{TO}_{\kappa_2}-\mathsf{TO}_{\kappa_1}\right) \tag{14}$$

The *break-even transaction cost* (BETC) is defined as the value of $\tau > 0$ that sets equation (14) to zero:

$$\mathsf{BETC}_{\kappa_1\to\kappa_2} = -\frac{\gamma}{4}\cdot\frac{\mathsf{PV}_{\kappa_2}-\mathsf{PV}_{\kappa_1}}{\mathsf{TO}_{\kappa_2}-\mathsf{TO}_{\kappa_1}}$$

[22] Defining the semi-leverage $\frac{1}{2}LV_{\kappa,t} \equiv -\sum_{\omega<0}\omega_{\kappa,j,t}$ and considering that the portfolio weights add to unity, it follows that: $\mathsf{TO}_\kappa = 1+\overline{LV}_\kappa$.

[23] For investors trading large volumes in equity markets, average markup transaction costs, as defined in Section 7.4, usually range between 3 to 10 basis points and average around 5 and 6 basis points.

8 Results

For all approaches introduced earlier, the OOS portfolio variances[24] PV are reported in Tables 1-4, the OOS certainty equivalent returns CEQ in Tables 5-8, the turnover TO in Tables 9-11, the OOS break-even transaction costs BETC in Tables 12-14. All statistical tests presented have been bootstrapped with 10,000 replications. Unless otherwise stated, the comparisons are statistically significant at any level.

8.1 VT Forecasts

Results for Volatility Timing (VT) approaches, which forecast the conditional variances but set the conditional correlation matrix to the identity matrix, are reported in Table 1 for all the variance specifications of Section 3.1 estimated by LS and QML. VT produces portfolio variances that are statistically less than those of the Naive portfolio and which range between -14.34% and -16.08%. LnGarch and InvGarch produce OOS portfolio variances that are smaller than those of Garch and Har. Table 5 reports the CEQ value of switching from Naive to VT. Overall, the switch is worth between 3.63 and 4.07 daily basis points (bps). Gains are positive for every variance specification, objective function and sub-period. Notice that turnover TO is 1 by construction for VT strategies as there is no short selling.

8.2 DCC Forecasts

Portfolio variances PV from the DCC modeling of the conditional correlation matrix are reported in Table 2. In each of the four panels, the gains with respect to the Naive strategy are significant for every variance specification, objective function and sub-period. The magnitude of the gains is also substantial, and ranges from a 40.31% to a 44.47% reduction in the portfolio variance. This is more than double the reduction achieved by the VT strategy. In fact, with respect to the best performing VT specification, gains from DCC modeling range between 28.88% and 33.84% in variance reduction. Overall there are no major PV differences between the Scalar- and

[24]In the literature on forecasting expected returns, there is consensus that the poor performance of theoretically optimal portfolio allocations is due to sizable *forecasting errors*. De Miguel *et al.* (2009) find that portfolio strategies incorporating short sale constraints perform much better than unconstrained policies trying to incorporate estimation or forecasting errors. Therefore, although we do not present the extended results here, we have investigated the performance of portfolios with short sale constraints as an indirect measure of the magnitude of potential forecasting errors. What we found is a deterioration of the resulting portfolio variances for all specifications considered ranging between +6.5% and +14%. Therefore, in our setting, benefits from short sale constraints, in terms of reduction of forecasting errors, are outbalanced by the portfolio's loss of efficiency.

Diagonal-DCC specifications. On the other hand, it does emerge that LnGarch LS delivers portfolio variances that are statistically smaller than those of the other variance models (the only exception is the InvGarch LS in the QML-estimated DCC specifications). In general, LS estimation produces better results than QML, and, as expected, the Diagonal parameterization of DCC does not add significantly to the forecasts of the scalar parameterization.

Table 6 reports the CEQ value of switching from VT to DCC modeling and forecasting. The switch is worth between 6.46 and 7.19 daily bps. Gains are positive and significant for every variance specification, objective function and sub-period. Average daily turnover TO is reported in Table 9 and ranges between 1.31 and 1.74.

The fact that DCC optimal portfolios have been constructed without taking into account transaction costs and turnover requires to take the BETC results, reported in Table 12, *cum grano salis*. DCC modeling is preferred to VT for transaction costs of less than 5.5 bps. However, if the investor's risk aversion γ is 2 or greater and transaction costs are no more than 10 bps, DCC is preferred to VT for any variance specification.

8.3 DCIC Forecasts

Since the estimation of DCIC has highlighted parameter estimates tending towards integrated processes, the constraints $a_i = 1 - b_i$ for every i were imposed to equation (4). Portfolio allocations from DCIC modeling, reported in Table 3, exhibit PV significant reductions between 40.61% and 44.31% with respect to the Naive strategy and between 29.33% and 33.65% with respect to the best performing VT. Again, the best allocations are achieved by the LnGarch LS modeling of the conditional variances. With respect to that of DCC, the PV are significantly smaller when both models are QML-estimated regardless of the conditional variance specification. On the other hand, when DCC and DCIC are LS-estimated, the PV of DCIC are significantly larger in 11/16 cases at 10%, 7/16 cases at 5% and 3/16 cases at 1%. These results highlight that if QML is the chosen method of estimation, DCIC is not only computationally more convenient but it may also generate better allocations than DCC. On the other hand, if LS is chosen, DCC is superior to DCIC. This finding confirms that DCIC needs an *ad hoc* LS objective function that may reconcile the weights assigned to the on- and off-diagonal elements of the correlation matrices in a more effective way than that adopted in equation (8). This being said, comparing the best performing DCC specification in each of the four panels of Table 2 with the corresponding DCIC, relative to the former, the latter never exhibits PV that are more than 1% larger.

The CEQ of Table 7 confirm that the differences between DCC and DCIC are relatively marginal. On the one hand, the most an investor is willing to pay is 0.07 daily bps, statistically significant, to switch from DCIC Diagonal LS + Garch QML to Diagonal DCC LS + Garch QML. On the other hand, the most an investor is willing to pay is 0.21 daily bps, statistically significant, to switch from Diagonal DCC QML + InvGarch QML to Diagonal DCIC QML + InvGarch QML. Average daily turnover TO is reported in Table 10 and ranges between 1.31 and 1.80 (similarly to DCC).

BETC of Table 13 show that, when LS-estimated, DCIC is never preferred to DCC due to higher portfolio variance PV and turnover TO. On the other hand, when QML-estimated, DCIC is preferred for low transaction costs. Since the break-even transaction costs of Table 13 need to be multiplied by γ, DCIC would be preferred to DCC (for most variance specifications) when $\gamma \geq 5$ and average transaction costs of 5-6 bps.

8.4 DCW Forecasts

Portfolio allocations from the direct dynamic modeling of the portfolio weights are reported in Table 4. PV reductions range between 43.97% and 44.72% with respect to the Naive strategy and between 33.24% and 34.13% with respect to VT. None of the DCW specifications of Table 4 exhibits a performance that is statistically different from the best DCC specification Scalar DCC LS + LnGarch LS. Furthermore, in the direct modeling of the weights, the objective function OP produces OOS variances that are smaller than those of LS and QML. Nevertheless, LS exhibits a more than reasonable performance with PV between 0.16% and 0.65% larger than those of OP.

Table 8 reports the CEQ value of switching from DCC to DCW. While no negative CEQ is statistically significant, with respect to QML estimation, the investor is willing to pay between 0.40 and 0.47 daily bps, statistically significant, to switch from DCC to DCW. Average daily turnover TO is reported in Table 11 and ranges between 1.21 and 1.72, slightly lower than that of DCC and DCIC.

BETC of Table 14 show that DCW is always preferred to DCC due to generally equal PV but lower TO. In fact, even for investors with risk aversion $\gamma = 10$, break-even transaction costs would be lower than 1 bps.

It must be emphasized that the DCW Diagonal LS of panel 2 in Table 4 is genuinely estimated equation-by-equation. Despite the fact that this data-set is best described by a scalar specification (cf. the relative performances of Diagonal and Scalar in Tables

2 and 3) the element-by-element modeling of the weights exhibits PV reductions of 44.01% when compared to Naive and 33.28% when compared to the best VT. PV is 0.83% and 0.54% larger than that of the best DCC and DCIC specifications, respectivelly, although none of the differences is statistically significant. Hence, the element-by-element DCW modeling is more than a valid alternative: it is as simple and computationally convenient as VT, its portfolio allocations are substantially superior to VT (while not statistically inferior to competing approaches like DCC). Most importantly, it is easily scalable to large cross-sectional dimensions M for which other approaches generally fail.

9 Conclusions

Evaluating the contribution of conditional second moments' modeling to optimal portfolio allocations, we find substantial improvements upon the simple Naive allocation when conditional variances are modeled and their forecasts incorporated in a VT strategy. We do find further striking improvements upon both the Naive and VT strategies from the addition of conditional correlations either in the DCC, DCIC or DCW forecasting models. Hence, the financial relevance of incorporating conditional second moments information of good quality in portfolio optimization problems.

With respect to conditional variance models considered, we find that LnGarch LS produces forecasts that are superior to those of the competing specifications, at any significance level. Similarly, we find that the Scalar DCC LS produces the best conditional correlation forecasts, among the DCC specifications considered.

As per motivations, when QML-estimated, the proposed DCIC is computationally more convenient than DCC and gives superior forecasts in terms of PV, CEQ and in many cases BETC. On the other hand, when LS-estimated, DCIC is never preferred to DCC. The superior performance of DCIC when estimated using the invariant QML objective function, suggests that this parameterization has the potential to perform better when an appropriate LS objective function is employed.

The proposed DCW, as computationally convenient as a simple volatility timing strategy VT, produces forecasts that are never statistically inferior to DCC in terms of PV and CEQ. Thanks to its generally lower turnover TO, in the presence of transaction costs, DCW is always preferred to DCC. Even for investors with risk-aversion of $\gamma = 10$, break-even transaction costs would be lower than 1 bps. Both Scalar- and Diagonal-DCW are very easily scalable to large cross-sectional dimensions for which other approaches generally fail. Furthermore, while both DCW and SCC bypass the *curse of dimensionality*, in optimal portfolio applications the former is computationally more convenient as its modeling dimension is *linear* in the cross-section, while the former, and all conditional correlation models, have modeling dimensions that are *quadratic* in the cross-section.

The definition of an *ad hoc* LS distance for the DCIC, the explicit inclusion of transaction costs in the investor's objective function and its implications for the various approaches presented, the evaluation of the proposed DCIC and DCW on more *size-heterogeneous* pools of assets and the combination of DCW with approaches that focus on the prediction of expected returns are extensions that we leave to future research.

A Appendix

A.1 Data Handling

For each trading day t, let $\{x_j\}_{j=1}^{J}$ be the collection of the $(M \times 1)$ return-vectors resulting from price-vectors synchronized according to Barndorff-Nielsen *et al.* (2011). Furthermore, let $\{\widetilde{x}_j\}_{j=1}^{\widetilde{J}}$ be the collection of return-vectors in the j-th bin of equally spaced 15 minute intervals. The daily realized kernel variance-covariance matrix is then computed as:

$$\widehat{\Omega} = \sum_{h=-l}^{l} k\left(\frac{h}{H}\right)\Gamma_h$$

where Γ_h is:

$$\Gamma_h = \begin{cases} \displaystyle\sum_{j=h+1}^{J} x_j x'_{j-h} & \text{if } h \geq 0 \\ \displaystyle\sum_{j=-h+1}^{J} x_{j+h} x'_j & \text{if } h < 0 \end{cases}$$

$k(x)$ is the Parzen kernel:

$$k(x) = \begin{cases} 1 - 6x^2 + 6x^3 & \text{if } x \in [0, 1/2] \\ 2(1-x)^3 & \text{if } x \in (1/2, 1] \\ 0 & \text{otherwise} \end{cases}$$

and H is given by:

$$H = \frac{1}{M}\sum_{i=1}^{M} 3.51 \cdot J^{3/5}\left(\frac{(2J)^{-1}\sum_{j=1}^{J} x_{i,j}^2}{\sum_{j=1}^{\widetilde{J}} \widetilde{x}_{i,j}^2}\right)^{2/5}$$

where $x_{i,j}$ and $\widetilde{x}_{i,j}$ are the i-th elements of the vectors x_j and $\widetilde{x}_j$, respectively, and $l = \min(H, J-1)$.

A.2 Gaussian Likelihood

Let $\{\varepsilon_{n,t}\}_{n=1}^{N}$ be the $(M \times 1)$ vectors of equally spaced, mean zero and serially uncorrelated intra-daily observations and $\widehat{\Omega}_t = \sum_{n=1}^{N} \varepsilon_{n,t}\varepsilon'_{n,t}$ the corresponding realized variance-covariance matrix. For simplicity, treat the intra-daily variance-covariance matrices as constant within each day: $\mathbb{V}[\varepsilon_{n,t}] = N^{-1} \cdot \Omega_t$ and $\mathbb{V}\left[\sum_{n=1}^{N} \varepsilon_{n,t}\right] = \Omega_t$. The concentrated Gaussian log-likelihood for observation $\varepsilon_{n,t}$ is:

$$\begin{aligned} l_{n,t} &= -\ln\left|N^{-1} \cdot \Omega_t\right| - N \cdot \varepsilon'_{n,t}\Omega_t^{-1}\varepsilon_{n,t} \\ &= M\ln(N) - \ln|\Omega_t| - N \cdot \varepsilon'_{n,t}\Omega_t^{-1}\varepsilon_{n,t} \end{aligned}$$

Dropping the constant term on the right-hand-side, the concentrated log-likelihood for the observations of day t is given by:

$$\begin{aligned} l_t &= -\sum_{n=1}^{N} \ln|\Omega_t| - N\sum_{n=1}^{N} \varepsilon'_{n,t}\Omega_t^{-1}\varepsilon_{n,t} \\ &= -N \cdot \ln|\Omega_t| - N \cdot \mathbb{TR}\left(\Omega_t^{-1}\widehat{\Omega}_t\right) \end{aligned}$$

Dropping the proportionality factor N, the concentrated log-likelihood for the sample of size T is then given by:

$$l = -\sum_{t=1}^{T}\left[\ln|\Omega_t| + \mathbb{TR}\left(\Omega_t^{-1}\widehat{\Omega}_t\right)\right] \tag{15}$$

The concentrated log-likelihood for the estimation of the conditional models of Sections 3.2 and 3.3 is obtained from equation (15) by replacing the conditional and realized variance-covariance matrices with the conditional and realized correlation matrices, respectively:

$$l = -\sum_{t=1}^{T}\left[\ln|R_t| + \mathbb{TR}\left(R_t^{-1}\widehat{R}_t\right)\right]$$

Setting $M = 1$ yields the univariate Gaussian log-likelihood used to estimate the conditional variance models of Section 3.1. While, setting $M = 2$ yields the bivariate Gaussian log-likelihood used to estimate the element-by-element specifications of the conditional inverse correlations of Section 3.3.

A.3 Transaction Costs Approximation

Recall equation (13) and let $\overline{\overline{R}}$ be the daily weighted average return over the entire time series and across all assets:

$$\overline{\overline{R}} \equiv \frac{\frac{1}{T\cdot M}\sum_{t=1}^{T}\sum_{j=1}^{M}|\omega_{\kappa,j,t}|\cdot R_{j,t}^{oc}}{\frac{1}{T\cdot M}\sum_{t=1}^{T}\sum_{j=1}^{M}|\omega_{\kappa,j,t}|}$$

Then:

$$\begin{aligned} \tau\cdot\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{M} R_{j,t}^{oc}|\omega_{\kappa,j,t}| &= \tau\overline{\overline{R}}\cdot\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{M}|\omega_{\kappa,j,t}| \\ &= \tau\overline{\overline{R}}\cdot \mathsf{TO}_{\kappa} \end{aligned}$$

from which it follows that the average transaction cost associated with model κ is given by:

$$\begin{aligned}\overline{TC}_{\kappa} &= 2\tau \cdot \mathsf{TO}_{\kappa} + \tau\overline{\overline{R}} \cdot \mathsf{TO}_{\kappa} \\ &\approx 2\tau \cdot \mathsf{TO}_{\kappa}\end{aligned}$$

given that $\overline{\overline{R}}$ is usually a very small number.

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Table 1:

Average out-of-sample daily variances PV of the Naive portfolio strategy and of the VT strategy for all variance and objective function specifications considered. The symbols *, ** and *** indicate that the VT variance is significantly smaller than the Naive variance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
Naive	0.765768	0.940376	0.337162	0.247769	0.265182	0.477073	0.505796
VT							
Garch LS	0.628512***	0.774085***	0.276035***	0.226469***	0.237077***	0.442673***	0.431006***
Garch QML	0.619003***	0.765655***	0.267558***	0.226375***	0.239009***	0.440675***	0.426580***
LnGarch LS	0.616889***	0.755255***	0.264938***	0.226838***	0.239349***	0.443525***	0.424664***
LnGarch QML	0.615938***	0.762678***	0.264855***	0.226517***	0.238980***	0.441084***	0.425210***
InvGarch LS	0.628599***	0.761852***	0.264861***	0.227169***	0.239974***	0.442630***	0.427720***
InvGarch QML	0.614299***	0.759060***	0.265133***	0.226861***	0.239441***	0.440931***	0.424487***
Har LS	0.629024***	0.776975***	0.277726***	0.228755***	0.242804***	0.443216***	0.433283***
Har QML	0.619207***	0.766770***	0.266976***	0.226444***	0.239520***	0.441176***	0.426884***

Table 2:

Average out-of-sample daily variances PV of the of the minimum variance portfolio constructed from the forecasts of the various variance, DCC and objective function specifications considered. The symbols *, ** and *** indicate that the DCC portfolio variance is significantly smaller than the VT variance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCC LS							
Garch LS	0.367726***	0.424391***	0.198676***	0.210328***	0.211125***	0.387286***	0.299998***
Garch QML	0.356986***	0.408238***	0.189633***	0.206708***	0.196900***	0.361714***	0.286776***
LnGarch LS	0.351059***	0.396345***	0.187109***	0.204369***	0.193208***	0.352605***	0.280859***
LnGarch QML	0.353260***	0.408446***	0.190171***	0.207688***	0.196711***	0.359867***	0.286102***
InvGarch LS	0.364467***	0.403361***	0.186760***	0.204402***	0.193623***	0.353503***	0.284436***
InvGarch QML	0.355923***	0.411099***	0.192500***	0.209378***	0.197347***	0.355478***	0.287034***
Har LS	0.363172***	0.424437***	0.199700***	0.211095***	0.206934***	0.376261***	0.297010***
Har QML	0.357097***	0.407351***	0.189949***	0.206387***	0.197345***	0.358907***	0.286252***
Scalar DCC QML							
Garch LS	0.369713***	0.430140***	0.197520***	0.209104***	0.210215***	0.392734***	0.301649***
Garch QML	0.361195***	0.419130***	0.190367***	0.208174***	0.199696***	0.370714***	0.291628***
LnGarch LS	0.355591***	0.405796***	0.187755***	0.206080***	0.196058***	0.366492***	0.286373***
LnGarch QML	0.357455***	0.418440***	0.190963***	0.209275***	0.199498***	0.368356***	0.290745***
InvGarch LS	0.366162***	0.409325***	0.186467***	0.204737***	0.194685***	0.361826***	0.287284***
InvGarch QML	0.359737***	0.420007***	0.193227***	0.210723***	0.200018***	0.364975***	0.291529***
Har LS	0.365841***	0.430374***	0.198447***	0.209929***	0.208272***	0.383793***	0.299521***
Har QML	0.360978***	0.418014***	0.190383***	0.207583***	0.199321***	0.368409***	0.290863***
Diagonal DCC LS							
Garch LS	0.366709***	0.424081***	0.198612***	0.210201***	0.210528***	0.386141***	0.299455***
Garch QML	0.356260***	0.409239***	0.189812***	0.206860***	0.196998***	0.360994***	0.286773***
LnGarch LS	0.350539***	0.397453***	0.187307***	0.204612***	0.193251***	0.352424***	0.281008***
LnGarch QML	0.352617***	0.409244***	0.190419***	0.207867***	0.196745***	0.359074***	0.286072***
InvGarch LS	0.362809***	0.403303***	0.186803***	0.204624***	0.193428***	0.352862***	0.284055***
InvGarch QML	0.355254***	0.411918***	0.192782***	0.209568***	0.197405***	0.355292***	0.287116***
Har LS	0.362485***	0.424110***	0.199625***	0.210947***	0.206844***	0.375209***	0.296613***
Har QML	0.356481***	0.408366***	0.190116***	0.206549***	0.197428***	0.358295***	0.286285***
Diagonal DCC QML							
Garch LS	0.369272***	0.431524***	0.197746***	0.209065***	0.209952***	0.393292***	0.301886***
Garch QML	0.361138***	0.421059***	0.190733***	0.208301***	0.199872***	0.371970***	0.292260***
LnGarch LS	0.355664***	0.407720***	0.188139***	0.206287***	0.196240***	0.368228***	0.287124***
LnGarch QML	0.357446***	0.420212***	0.191359***	0.209426***	0.199647***	0.369680***	0.291376***
InvGarch LS	0.365886***	0.410400***	0.186781***	0.204979***	0.194751***	0.363295***	0.287766***
InvGarch QML	0.359662***	0.421682***	0.193613***	0.210865***	0.200168***	0.366557***	0.292172***
Har LS	0.365420***	0.431559***	0.198703***	0.209950***	0.208300***	0.384579***	0.299830***
Har QML	0.360869***	0.419999***	0.190753***	0.207740***	0.199563***	0.369793***	0.291534***

Table 3:

Average out-of-sample daily variances PV of the of the minimum variance portfolio constructed from the forecasts of the various variance, DCIC and objective function specifications considered. The symbols *, ** and *** indicate that the DCIC portfolio variance is significantly smaller than the DCC variance at the 10%, 5% and 1% level, respectively. Additionally, $_{*}$, $_{**}$ and $_{***}$ indicate that the DCIC portfolio variance is significantly larger than the DCC portfolio variance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCIC LS							
Garch LS	0.366000	0.425904	0.198542	0.209660	0.209315***	0.392444$_{***}$	0.300385
Garch QML	0.357886	0.410823$_{***}$	0.189402	0.205578**	0.197326	0.366907$_{***}$	0.288065$_{***}$
LnGarch LS	0.351207	0.397673	0.186750	0.203383**	0.193665	0.356859$_{***}$	0.281665
LnGarch QML	0.353524	0.410810$_{**}$	0.189625	0.206423***	0.1196752	0.364830$_{***}$	0.287071$_{*}$
InvGarch LS	0.362837	0.405922$_{***}$	0.186490	0.203431**	0.193684	0.356799$_{***}$	0.284943
InvGarch QML	0.355967	0.413273$_{**}$	0.191836*	0.208107***	0.197199	0.359600$_{***}$	0.287743
Har LS	0.361954	0.425950	0.199337	0.210425	0.206765	0.381940$_{***}$	0.297803
Har QML	0.357785	0.410086$_{***}$	0.189640	0.205234***	0.197920	0.364073$_{***}$	0.287535$_{**}$
Scalar DCIC QML							
Garch LS	0.367989	0.428706	0.197146	0.207095***	0.206635***	0.394148	0.300361***
Garch QML	0.361513	0.417596	0.189379**	0.204439***	0.197602***	0.370765	0.290296***
LnGarch LS	0.355462	0.403520	0.186721***	0.202363***	0.193981***	0.365098	0.284600***
LnGarch QML	0.357409	0.416650	0.189551***	0.205257***	0.196872***	0.368228	0.289074***
InvGarch LS	0.364886	0.408962	0.185898	0.201489***	0.192851***	0.361269	0.285975***
InvGarch QML	0.359543	0.418143	0.191665***	0.206692***	0.197210***	0.364241	0.289662***
Har LS	0.365009	0.428978	0.197839	0.207710***	0.205756***	0.385038	0.298464***
Har QML	0.361451	0.416617	0.189320***	0.203873***	0.197604***	0.368613	0.289660***
Diagonal DCIC LS							
Garch LS	0.366905	0.425841	0.198188	0.209374*	0.208881***	0.392706$_{***}$	0.300390$_{*}$
Garch QML	0.358719$_{***}$	0.411046	0.189136	0.205444***	0.197280	0.367210$_{***}$	0.288218$_{***}$
LnGarch LS	0.351915$_{*}$	0.397709	0.186495**	0.203269***	0.193693	0.357760$_{***}$	0.281883$_{*}$
LnGarch QML	0.354217$_{***}$	0.411020	0.189356***	0.206331***	0.196700	0.365084$_{***}$	0.287196$_{**}$
InvGarch LS	0.363445	0.405876$_{**}$	0.186149*	0.203256***	0.193548	0.357240$_{***}$	0.285002$_{**}$
InvGarch QML	0.356416	0.413493	0.191575***	0.208019***	0.197168	0.360232$_{***}$	0.287897$_{*}$
Har LS	0.362402	0.425743	0.198966*	0.210109*	0.206761	0.382384$_{***}$	0.297802$_{**}$
Har QML	0.358035	0.410222	0.189361*	0.205084***	0.197864	0.364602$_{***}$	0.287607$_{***}$
Diagonal DCIC QML							
Garch LS	0.368192	0.428712*	0.197098	0.206989***	0.206437***	0.394091	0.300328***
Garch QML	0.361709	0.417849*	0.189357***	0.204332***	0.197508***	0.370800	0.290340***
LnGarch LS	0.355667	0.403751**	0.186794***	0.202273***	0.193919***	0.365201**	0.284662***
LnGarch QML	0.357603	0.416884*	0.189527***	0.205151***	0.196782***	0.368268	0.289115***
InvGarch LS	0.363445	0.405876	0.186149	0.203256**	0.193548	0.357240***	0.285002***
InvGarch QML	0.356416***	0.413493**	0.191575***	0.208019***	0.197168***	0.360232***	0.287897***
Har LS	0.362402**	0.425743	0.198966	0.210109	0.206761	0.382384	0.297802***
Har QML	0.361559	0.416865	0.189295***	0.203767***	0.197508***	0.368662	0.289690***

Table 4:

Average out-of-sample daily variances PV of the of the minimum variance portfolio constructed from the direct modeling and forecasting of the portfolio weights DCW. Each specification has been estimated by minimizing the in-sample-portfolio variance (OP), by least-squares (LS) and by quasi maximum likelihood (QML). The symbols *, ** and *** indicate that the DCW portfolio variance is significantly smaller than the DCC variance at the 10%, 5% and 1% level, respectively. Additionally, $_{*}$, $_{**}$ and $_{***}$ indicate that the DCW portfolio variance is significantly larger than the DCC portfolio variance at the 10%, 5% and 1% level, respectively. When possible, comparisons are conducted with the best corresponding DCC specification. First and Third Panels: comparisons with Scalar DCC § + LnGarch §. Second Panel: comparisons with Diagonal DCC § + LnGarch §. In all panels, § is LS for OP and LS and is QML otherwise.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCW							
OP	0.357367	0.395033	0.183606***	0.201635**	0.190428***	0.350648	0.279867
LS	0.364858$_{**}$	0.400907$_{**}$	0.181594***	0.199397***	0.190334***	0.352549	0.281692
QML	0.365464	0.401505***	0.181564***	0.199455***	0.190378***	0.352731***	0.281935***
Diagonal DCW							
OP	0.365437	0.396500	0.182956***	0.201835**	0.191241**	0.357957$_{***}$	0.282737
LS	0.368602$_{***}$	0.404762$_{**}$	0.180981***	0.200283***	0.190229***	0.353792	0.283197
QML	0.369197	0.405055***	0.180980***	0.200317***	0.190279***	0.353985***	0.283391**
Scalar DCW $(2, 1)$							
OP	0.358137	0.393704	0.183129***	0.200998***	0.189962***	0.351198	0.279602
LS	0.365179	0.399066	0.181174***	0.198588***	0.189743***	0.353046	0.281217
QML	0.365736	0.399615***	0.181147***	0.198633***	0.189789***	0.353210***	0.281440***

Table 5:

Average out-of-sample daily certainty equivalent CEQ, expressed in *basis points*, of the VT strategy with respect to the Naive portfolio strategy. CEQ are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. The symbols *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
VT							
Garch LS	6.86***	8.31***	3.06***	1.07***	1.41***	1.72***	3.74***
Garch QML	7.34***	8.74***	3.48***	1.07***	1.31***	1.82***	3.96***
LnGarch LS	7.44***	9.26***	3.61***	1.05***	1.29***	1.68***	4.06***
LnGarch QML	7.49***	8.88***	3.62***	1.06***	1.31***	1.80***	4.03***
InvGarch LS	6.86***	8.93***	3.62***	1.03***	1.26***	1.72***	3.90***
InvGarch QML	7.57***	9.07***	3.60***	1.05***	1.29***	1.81***	4.07***
Har LS	6.84***	8.17***	2.97***	0.95***	1.12***	1.69***	3.63***
Har QML	7.33***	8.68***	3.51***	1.07***	1.28***	1.79***	3.95***

Table 6:

Average out-of-sample daily certainty equivalent CEQ, expressed in *basis points*, of the DCC minimum variance portfolio with respect to the VT strategy. CEQ are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. The symbols *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCC LS							
Garch LS	13.04***	17.48***	3.87***	0.81***	1.30***	2.77***	6.55***
Garch QML	13.10***	17.87***	3.90***	0.98***	2.11***	3.95***	6.99***
LnGarch LS	13.29***	17.95***	3.89***	1.12***	2.31***	4.55***	7.19***
LnGarch QML	13.13***	17.71***	3.73***	0.94***	2.11***	4.06***	6.96***
InvGarch LS	13.21***	17.92***	3.91***	1.14***	2.32***	4.46***	7.16***
InvGarch QML	12.92***	17.40***	3.63***	0.87***	2.10***	4.27***	6.87***
Har LS	13.29***	17.63***	3.90***	0.88***	1.79***	3.35***	6.81***
Har QML	13.11***	17.97***	3.85***	1.00***	2.11***	4.11***	7.03***
Scalar DCC QML							
Garch LS	12.94***	17.20***	3.93***	0.87***	1.34***	2.50***	6.47***
Garch QML	12.89***	17.33***	3.86***	0.91***	1.97***	3.50***	6.75***
LnGarch LS	13.06***	17.47***	3.86***	1.04***	2.16***	3.85***	6.91***
LnGarch QML	12.92***	17.21***	3.69***	0.86***	1.97***	3.64***	6.72***
InvGarch LS	13.12***	17.63***	3.92***	1.12***	2.26***	4.04***	7.02***
InvGarch QML	12.73***	16.95***	3.60***	0.81***	1.97***	3.80***	6.65***
Har LS	13.16***	17.33***	3.96***	0.94***	1.73***	2.97***	6.69***
Har QML	12.91***	17.44***	3.83***	0.94***	2.01***	3.64***	6.80***
Diagonal DCC LS							
Garch LS	13.09***	17.50***	3.87***	0.81***	1.33***	2.83***	6.58***
Garch QML	13.14***	17.82***	3.89***	0.98***	2.10***	3.98***	6.99***
LnGarch LS	13.32***	17.89***	3.88***	1.11***	2.30***	4.56***	7.18***
LnGarch QML	13.17***	17.67***	3.72***	0.93***	2.11***	4.10***	6.96***
InvGarch LS	13.29***	17.93***	3.90***	1.13***	2.33***	4.49***	7.18***
InvGarch QML	12.95***	17.36***	3.62***	0.86***	2.10***	4.28***	6.87***
Har LS	13.33***	17.64***	3.91***	0.89***	1.80***	3.40***	6.83***
Har QML	13.14***	17.92***	3.84***	0.99***	2.10***	4.14***	7.03***
Diagonal DCC QML							
Garch LS	12.96***	17.13***	3.91***	0.87***	1.36***	2.47***	6.46***
Garch QML	12.89***	17.23***	3.84***	0.90***	1.96***	3.44***	6.72***
LnGarch LS	13.06***	17.38***	3.84***	1.03***	2.16***	3.76***	6.88***
LnGarch QML	12.92***	17.12***	3.67***	0.85***	1.97***	3.57***	6.69***
InvGarch LS	13.14***	17.57***	3.90***	1.11***	2.26***	3.97***	7.00***
InvGarch QML	12.73***	16.87***	3.58***	0.80***	1.96***	3.72***	6.62***
Har LS	13.18***	17.27***	3.95***	0.94***	1.73***	2.93***	6.67***
Har QML	12.92***	17.34***	3.81***	0.94***	2.00***	3.57***	6.77***

Table 7:

Average out-of-sample daily certainty equivalent CEQ, expressed in *basis points*, of the DCIC minimum variance portfolio with respect to the DCC minimum variance portfolio. CEQ are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. The symbols *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCIC LS							
Garch LS	0.09	-0.08	0.01	0.03	0.09***	-0.26***	-0.02
Garch QML	-0.04	-0.13***	0.01	0.06**	-0.02	-0.26***	-0.06***
LnGarch LS	-0.01	-0.07	0.02	0.05**	-0.02	-0.21***	-0.04
LnGarch QML	-0.01	-0.12**	0.03	0.06***	-0.00	-0.25***	-0.05*
InvGarch LS	0.08	-0.13***	0.01	0.05**	-0.00	-0.16***	-0.02
InvGarch QML	-0.00	-0.11**	0.03*	0.06***	0.01	-0.21***	-0.04
Har LS	0.06	-0.08	0.02	0.03	0.01	-0.28***	-0.04
Har QML	-0.03	-0.14***	0.02	0.06***	-0.03	-0.25***	-0.06**
Scalar DCIC QML							
Garch LS	0.09	0.07	0.02	0.10***	0.18***	-0.07	0.06***
Garch QML	-0.02	0.08	0.05**	0.19***	0.10***	-0.00	0.07***
LnGarch LS	0.01	0.11	0.05***	0.19***	0.10***	0.07	0.09***
LnGarch QML	0.00	0.09	0.07***	0.20***	0.13***	0.01	0.08***
InvGarch LS	0.06	0.02	0.03	0.16***	0.09***	0.03	0.07***
InvGarch QML	0.01	0.09	0.08***	0.20***	0.14***	0.04	0.09***
Har LS	0.04	0.07	0.03	0.11***	0.13***	-0.06	0.05***
Har QML	-0.02	0.07	0.05***	0.19***	0.09***	-0.01	0.06***
Diagonal DCIC LS							
Garch LS	-0.01	-0.09	0.02	0.04*	0.08***	-0.33***	-0.05*
Garch QML	-0.12***	-0.09	0.03	0.07***	-0.01	-0.31***	-0.07***
LnGarch LS	-0.07*	-0.01	0.04**	0.07***	-0.02	-0.27***	-0.04*
LnGarch QML	-0.08***	-0.09	0.05***	0.07***	0.00	-0.30***	-0.06*
InvGarch LS	-0.03	-0.13**	0.03*	0.07***	-0.01	-0.22***	-0.05**
InvGarch QML	-0.06	-0.08	0.06***	0.08***	0.01	-0.24***	-0.04*
Har LS	0.00	-0.08	0.03*	0.04*	0.00	-0.36***	-0.06**
Har QML	-0.08	-0.09	0.04*	0.07***	-0.02	-0.33***	-0.07***
Diagonal DCIC QML							
Garch LS	0.05	0.14*	0.03	0.10***	0.18***	-0.04	0.08***
Garch QML	-0.03	0.16*	0.07***	0.20***	0.12***	0.06	0.10***
LnGarch LS	-0.00	0.20*	0.07***	0.20***	0.12***	0.15**	0.12***
LnGarch QML	-0.01	0.17*	0.09***	0.21***	0.14***	0.07	0.11***
InvGarch LS	0.12	0.23	0.03	0.09**	0.06	0.30***	0.14***
InvGarch QML	0.16***	0.41**	0.10***	0.14***	0.15***	0.32***	0.21***
Har LS	0.15**	0.29	-0.01	-0.01	0.08	0.11	0.10***
Har QML	-0.03	0.16	0.07***	0.20***	0.10***	0.06	0.09***

Table 8:

Average out-of-sample daily certainty equivalent CEQ, expressed in *basis points*, of the DCW minimum variance portfolio with respect to the DCC minimum variance portfolio. CEQ are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. The symbols *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively. When possible, comparisons are conducted with the best corresponding DCC specification. First and Third Panels: comparisons with Scalar DCC § + LnGarch §. Second Panel: comparisons with Diagonal DCC § + LnGarch §. In all panels, § is LS for OP and LS and is QML otherwise.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCW							
OP	-0.32	0.07	0.18^{***}	0.14^{**}	0.14^{***}	0.10	0.05
LS	-0.69^{**}	-0.23^{**}	0.28^{***}	0.25^{***}	0.14^{***}	0.00	-0.04
QML	-0.40	0.85^{***}	0.47^{***}	0.49^{***}	0.46^{***}	0.78^{***}	0.44^{***}
Diagonal DCW							
OP	-0.74	0.05	0.22^{***}	0.14^{**}	0.10^{**}	-0.28^{***}	-0.09
LS	-0.90^{***}	-0.37^{**}	0.32^{***}	0.22^{***}	0.15^{***}	-0.07	-0.11
QML	-0.59	0.76^{***}	0.52^{***}	0.46^{***}	0.47^{***}	0.78^{***}	0.40^{**}
Scalar DCW $(2,1)$							
OP	-0.35	0.13	0.20^{***}	0.17^{***}	0.16^{***}	0.07	0.06
LS	-0.71	-0.14	0.30^{***}	0.29^{***}	0.17^{***}	-0.02	-0.02
QML	-0.41	0.94^{***}	0.49^{***}	0.53^{***}	0.49^{***}	0.76^{***}	0.47^{***}

Table 9:

Average daily out-of-sample daily turnover TO of the minimum variance portfolio constructed from the forecasts of the various variance, DCC and objective function specifications considered.

Model	2010	2011	2012	2013	2014	2015	All
			Scalar DCC LS				
Garch LS	1.67	1.73	1.49	1.44	1.38	1.69	1.57
Garch QML	1.71	1.72	1.49	1.43	1.38	1.67	1.56
LnGarch LS	1.70	1.73	1.49	1.41	1.36	1.61	1.55
LnGarch QML	1.72	1.73	1.50	1.44	1.39	1.67	1.57
InvGarch LS	1.70	1.74	1.49	1.39	1.35	1.62	1.55
InvGarch QML	1.73	1.74	1.51	1.45	1.41	1.68	1.59
Har LS	1.68	1.74	1.49	1.43	1.47	1.69	1.59
Har QML	1.71	1.72	1.48	1.43	1.38	1.67	1.57
			Scalar DCC QML				
Garch LS	1.61	1.63	1.44	1.41	1.35	1.59	1.51
Garch QML	1.66	1.63	1.45	1.43	1.35	1.59	1.52
LnGarch LS	1.65	1.64	1.46	1.41	1.33	1.55	1.51
LnGarch QML	1.67	1.64	1.47	1.44	1.36	1.60	1.53
InvGarch LS	1.65	1.65	1.46	1.38	1.31	1.53	1.50
InvGarch QML	1.68	1.66	1.48	1.45	1.39	1.61	1.54
Har LS	1.62	1.64	1.44	1.40	1.43	1.60	1.52
Har QML	1.65	1.63	1.45	1.42	1.34	1.59	1.52
			Diagonal DCC LS				
Garch LS	1.67	1.73	1.49	1.44	1.39	1.69	1.57
Garch QML	1.71	1.72	1.49	1.43	1.39	1.66	1.57
LnGarch LS	1.70	1.72	1.49	1.41	1.36	1.61	1.55
LnGarch QML	1.72	1.72	1.50	1.44	1.40	1.67	1.57
InvGarch LS	1.70	1.74	1.49	1.40	1.35	1.61	1.55
InvGarch QML	1.73	1.74	1.51	1.45	1.42	1.68	1.59
Har LS	1.69	1.73	1.49	1.44	1.48	1.69	1.59
Har QML	1.71	1.72	1.49	1.44	1.39	1.67	1.57
			Diagonal DCC QML				
Garch LS	1.61	1.63	1.44	1.42	1.35	1.58	1.50
Garch QML	1.66	1.63	1.45	1.43	1.36	1.58	1.52
LnGarch LS	1.65	1.64	1.46	1.41	1.34	1.53	1.51
LnGarch QML	1.67	1.64	1.47	1.44	1.37	1.58	1.53
InvGarch LS	1.65	1.65	1.46	1.38	1.31	1.52	1.49
InvGarch QML	1.68	1.65	1.48	1.45	1.39	1.59	1.54
Har LS	1.62	1.63	1.44	1.41	1.43	1.58	1.52
Har QML	1.65	1.63	1.45	1.42	1.35	1.58	1.51

Table 10:

Average daily out-of-sample daily turnover TO of the minimum variance portfolio constructed from the forecasts of the various variance, DCIC and objective function specifications considered.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCIC LS							
Garch LS	1.72	1.79	1.48	1.42	1.38	1.78	1.60
Garch QML	1.75	1.77	1.48	1.41	1.38	1.76	1.59
LnGarch LS	1.75	1.77	1.48	1.39	1.36	1.71	1.58
LnGarch QML	1.76	1.78	1.48	1.41	1.38	1.76	1.60
InvGarch LS	1.75	1.79	1.48	1.37	1.35	1.71	1.58
InvGarch QML	1.77	1.78	1.50	1.43	1.40	1.77	1.61
Har LS	1.73	1.80	1.48	1.42	1.46	1.79	1.61
Har QML	1.75	1.78	1.47	1.41	1.38	1.77	1.59
Scalar DCIC QML							
Garch LS	1.67	1.72	1.42	1.37	1.33	1.70	1.54
Garch QML	1.71	1.70	1.43	1.37	1.33	1.69	1.54
LnGarch LS	1.70	1.70	1.43	1.35	1.31	1.65	1.52
LnGarch QML	1.72	1.71	1.44	1.38	1.33	1.70	1.55
InvGarch LS	1.70	1.72	1.43	1.32	1.29	1.64	1.52
InvGarch QML	1.73	1.72	1.46	1.39	1.35	1.70	1.56
Har LS	1.68	1.72	1.42	1.36	1.40	1.71	1.55
Har QML	1.70	1.70	1.43	1.37	1.32	1.69	1.54
Diagonal DCIC LS							
Garch LS	1.71	1.79	1.48	1.41	1.38	1.78	1.59
Garch QML	1.75	1.77	1.47	1.41	1.38	1.76	1.59
LnGarch LS	1.74	1.77	1.47	1.39	1.36	1.71	1.57
LnGarch QML	1.76	1.77	1.48	1.41	1.38	1.77	1.60
InvGarch LS	1.75	1.79	1.48	1.37	1.35	1.71	1.57
InvGarch QML	1.77	1.78	1.49	1.43	1.40	1.77	1.61
Har LS	1.73	1.79	1.48	1.42	1.46	1.79	1.61
Har QML	1.75	1.77	1.47	1.41	1.38	1.77	1.59
Diagonal DCIC QML							
Garch LS	1.66	1.71	1.42	1.37	1.33	1.70	1.53
Garch QML	1.70	1.70	1.43	1.37	1.33	1.69	1.54
LnGarch LS	1.69	1.70	1.43	1.35	1.31	1.65	1.52
LnGarch QML	1.71	1.70	1.44	1.38	1.33	1.69	1.54
InvGarch LS	1.70	1.72	1.43	1.32	1.29	1.63	1.52
InvGarch QML	1.72	1.71	1.45	1.39	1.35	1.70	1.55
Har LS	1.68	1.71	1.42	1.36	1.40	1.71	1.55
Har QML	1.70	1.70	1.43	1.37	1.32	1.69	1.53

Table 11:

Average daily out-of-sample daily turnover TO of the minimum variance portfolio constructed from the direct modeling and forecasting of the portfolio weights DCW. Each specification was estimated by minimizing the in-sample-portfolio variance (OP), by least-squares (LS) and by quasi maximum likelihood (QML).

ObjFun	2010	2011	2012	2013	2014	2015	All
Scalar DCW							
OP	1.70	1.72	1.43	1.34	1.32	1.66	1.53
LS	1.54	1.57	1.35	1.26	1.24	1.50	1.41
QML	1.53	1.57	1.35	1.26	1.24	1.49	1.41
Diagonal DCW							
OP	1.66	1.67	1.40	1.33	1.24	1.47	1.46
LS	1.49	1.55	1.34	1.27	1.21	1.42	1.38
QML	1.49	1.55	1.34	1.27	1.21	1.42	1.38
Scalar DCW $(2, 1)$							
OP	1.71	1.72	1.43	1.34	1.32	1.66	1.53
LS	1.55	1.58	1.35	1.26	1.24	1.51	1.41
QML	1.54	1.57	1.35	1.25	1.24	1.50	1.41

Table 12:

Average out-of-sample daily break-even transaction costs BETC, expressed in *basis points*, of the DCC minimum variance portfolio with respect to the VT minimum variance portfolio. BETC are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. $<$ and $>$ define the range of transaction costs for which DCC is preferred to VT.

Model	2010	2011	2012	2013	2014	2015	All
Scalar DCC LS							
Garch LS	<9.73	<11.95	<3.94	<0.92	<1.69	<2.01	<5.78
Garch QML	<9.29	<12.39	<4.01	<1.15	<2.77	<2.96	<6.19
LnGarch LS	<9.46	<12.37	<3.95	<1.38	<3.22	<3.71	<6.54
LnGarch QML	<9.43	<12.62	<3.94	<1.36	<3.20	<3.61	<6.56
InvGarch LS	<9.38	<12.04	<3.95	<1.45	<3.33	<3.60	<6.51
InvGarch QML	<8.86	<11.79	<3.55	<0.97	<2.56	<3.13	<5.85
Har LS	<9.72	<11.94	<3.96	<1.02	<1.89	<2.41	<5.81
Har QML	<9.26	<12.44	<3.97	<1.16	<2.77	<3.06	<6.21
Scalar DCC QML							
Garch LS	<10.62	<13.63	<4.47	<1.05	<1.93	<2.10	<6.39
Garch QML	<9.82	<13.66	<4.26	<1.08	<2.80	<2.95	<6.50
LnGarch LS	<10.01	<13.64	<4.18	<1.27	<3.24	<3.52	<6.81
LnGarch QML	<9.97	<13.93	<4.18	<1.25	<3.21	<3.41	<6.84
InvGarch LS	<10.15	<13.53	<4.29	<1.48	<3.64	<3.79	<7.08
InvGarch QML	<9.33	<12.91	<3.72	<0.89	<2.56	<3.13	<6.10
Har LS	<10.56	<13.64	<4.52	<1.18	<2.01	<2.47	<6.41
Har QML	<9.86	<13.74	<4.26	<1.12	<2.93	<3.07	<6.59
Diagonal DCC LS							
Garch LS	<9.73	<11.98	<3.94	<0.92	<1.71	<2.06	<5.78
Garch QML	<9.29	<12.39	<3.99	<1.12	<2.71	<3.02	<6.17
LnGarch LS	<9.46	<12.37	<3.92	<1.35	<3.16	<3.76	<6.52
LnGarch QML	<9.42	<12.62	<3.92	<1.33	<3.14	<3.66	<6.54
InvGarch LS	<9.43	<12.10	<3.95	<1.42	<3.31	<3.68	<6.53
InvGarch QML	<8.86	<11.80	<3.52	<0.95	<2.51	<3.16	<5.84
Har LS	<9.71	<12.00	<3.97	<1.01	<1.87	<2.47	<5.81
Har QML	<9.26	<12.45	<3.95	<1.14	<2.70	<3.11	<6.19
Diagonal DCC QML							
Garch LS	<10.63	<13.64	<4.46	<1.04	<1.95	<2.14	<6.42
Garch QML	<9.80	<13.67	<4.23	<1.05	<2.75	<2.98	<6.49
LnGarch LS	<9.99	<13.66	<4.15	<1.25	<3.19	<3.52	<6.80
LnGarch QML	<9.95	<13.95	<4.15	<1.23	<3.17	<3.41	<6.82
InvGarch LS	<10.15	<13.61	<4.27	<1.45	<3.60	<3.82	<7.08
InvGarch QML	<9.34	<12.93	<3.69	<0.88	<2.53	<3.14	<6.10
Har LS	<10.59	<13.70	<4.50	<1.16	<2.00	<2.52	<6.43
Har QML	<9.86	<13.77	<4.23	<1.10	<2.87	<3.09	<6.58

Table 13:

Average out-of-sample daily break-even transaction costs BETC, expressed in *basis points*, of the DCIC minimum variance portfolio with respect to the DCC minimum variance portfolio. BETC are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. $<$ and $>$ define the range of transaction costs for which DCIC is preferred to DCC. The entry A (N) indicates that DCIC is preferred to DCC for Any (No) value of τ.

Model	2010	2011	2012	2013	2014	2015	All
			Scalar DCIC LS				
Garch LS	<0.91	N	A	A	A	N	N
Garch QML	N	N	A	A	>6.07	N	N
LnGarch LS	N	N	A	A	N	N	N
LnGarch QML	N	N	A	A	>0.21	N	N
InvGarch LS	<0.84	N	A	A	N	N	N
InvGarch QML	N	N	A	A	A	N	N
Har LS	<0.64	N	A	A	A	N	N
Har QML	N	N	A	A	>11.47	N	N
			Scalar DCIC QML				
Garch LS	<0.76	<0.41	A	A	A	N	<1.08
Garch QML	N	<0.55	A	A	A	N	<1.72
LnGarch LS	<0.07	<0.90	A	A	A	<0.34	<2.62
LnGarch QML	<0.02	<0.67	A	A	A	<0.03	<2.65
InvGarch LS	<0.57	<0.13	A	A	A	<0.13	<1.46
InvGarch QML	<0.11	<0.74	A	A	A	<0.20	<3.92
Har LS	<0.35	<0.41	A	A	A	N	<0.89
Har QML	N	<0.50	A	A	A	N	<1.44
			Diagonal DCIC LS				
Garch LS	N	N	A	A	A	N	N
Garch QML	N	N	A	A	>0.62	N	N
LnGarch LS	N	N	A	A	>1.66	N	N
LnGarch QML	N	N	A	A	A	N	N
InvGarch LS	N	N	A	A	>0.58	N	N
InvGarch QML	N	N	A	A	A	N	N
Har LS	<0.05	N	A	A	A	N	N
Har QML	N	N	A	A	>0.85	N	N
			Diagonal DCIC QML				
Garch LS	<0.52	<0.84	A	A	A	N	<1.32
Garch QML	N	<1.18	A	A	A	<0.26	<2.64
LnGarch LS	N	<1.61	A	A	A	<0.68	<4.02
LnGarch QML	N	<1.29	A	A	A	<0.32	<3.85
InvGarch LS	<1.20	<1.60	A	A	A	<1.30	<3.24
InvGarch QML	<1.97	<3.37	A	A	A	<1.54	<9.68
Har LS	<1.34	<1.73	>0.35	>0.09	A	<0.42	<1.73
Har QML	N	<1.14	A	A	A	<0.25	<2.30

Table 14:

Average out-of-sample daily break-even transaction costs BETC, expressed in *basis points*, of the DCW minimum variance portfolio with respect to the DCC minimum variance portfolio. BETC are calculated for a risk-aversion coefficient of $\gamma = 1$ and may be computed for different values of γ by simple multiplication. $<$ and $>$ define the range of transaction costs for which DCW is preferred to DCC. The entry A (N) indicates that DCW is preferred to DCC for Any (No) value of τ.

Model	2010	2011	2012	2013	2014	2015	All
			Scalar DCW				
OP	>0.22	A	A	A	A	A	A
LS	>0.49	>0.16	A	A	A	A	>0.04
QML	>0.30	A	A	A	A	A	A
			Diagonal DCW				
OP	>0.53	A	A	A	A	>0.23	>0.08
LS	>0.64	>0.25	A	A	A	>0.06	>0.10
QML	>0.44	A	A	A	A	A	A
			Scalar DCW $(2,1)$				
OP	>0.25	A	A	A	A	A	A
LS	>0.50	>0.09	A	A	A	>0.02	>0.02
QML	>0.31	A	A	A	A	A	A